WILL Part I (addition)

Closed Algebraic System of Relational Orbital Mechanics (R.O.M.)

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October 2025

Desmos Project

R.O.M.

R.O.M. does not describe how a body moves under forces; it classifies the algebraically allowed relational states of a bound system.

$$\boxed{\kappa^2 = 1 - \frac{1}{(1+z_k)^2}} \quad (z_k = \text{gravitational redshift})$$

$$\boxed{\beta^2 = 1 - \frac{1}{(1+z_b)^2}} \quad (z_b = \text{transverse Doppler shift})$$

Observational Z Inputs

$Z_{sys}(o) = (1+z_{ko}(o))(1+z_{bo}(o)) = \tau_{Wo}(o)^{-1}$ (product of gravitational red shift and transverse Doppler shift)
$\tau_{Wo}(o) = \kappa_{Xo}(o) \cdot \beta_{Yo}(o) = (Z_{sys}(o))^{-1}$ (product of projectinal phase factors on S^1 and S^2 carriers)
$z_k = \frac{1}{\kappa_X} - 1$ (gravitational redshift)
$z_b = \frac{1}{\beta_Y} - 1$ (transverse Doppler shift)
$z_{ko} = \frac{1}{\kappa_{Xo}} - 1$ (redshift at phase o)
$z_{bo} = \frac{1}{\beta_{Yo}} - 1$ (transverse Doppler shift at phase o)

Global System Parameters (Fixed for the Orbit)

$\kappa = \sqrt{\frac{R_s}{a}} = \sqrt{\frac{\rho}{\rho_{max}}} = \sqrt{\kappa_p^2(1-e)} = \sqrt{4W} = \sqrt{2(\kappa_o(o)^2 - \beta_o(o)^2)} = \sqrt{\frac{1}{2}(3 - \sqrt{1 + 8\tau_{Wo}(O_o)^2})} = \sqrt{1-(1+z_k)^{-2}}$ (potential projection at semi-major axis)
$\kappa_X = \cos(\theta_2) = \sqrt{1-\kappa^2}$ (gravitational phase factor)
$\beta = \frac{\kappa}{\sqrt{2}} = \beta_p\sqrt{\frac{1-e}{1+e}} = \sqrt{2W} = \sqrt{(\kappa_o(o)^2 - \beta_o(o)^2)} = \sqrt{\kappa_o^2 - \frac{\kappa_o^2}{2} \cdot (1 + (\frac{1}{\delta_o(o)} - 1))} =$

*This work is archived on Zenodo: ¸

 DOI: 10.5281/zenodo.17115270

$\beta_o(o)\frac{\sqrt{1-e^2}}{\sqrt{1+e^2+2\cdot e\cdot\cos(o)}} = \sqrt{1-(1+z_b)^{-2}}$ (kinetic projection on semi major axis)
$\beta_Y = \sin(\theta_1) = \sqrt{1-\beta^2}$ (relativistic phase factor)
$\beta_{int} = \frac{\beta}{\sqrt{1-e^2}}$ (interior kinetic projection)
$\tau_W = \kappa_X \beta_Y$ (relational spacetime factor)
$Q = \sqrt{\kappa^2+\beta^2} = \sqrt{\frac{3}{2}}\ \kappa = \sqrt{3}\ \beta$ (total relational shift (magnitude of state difference))
$R_s = \kappa^2 a = \frac{2Gm_0}{c^2} = \frac{\Delta_{to}(o)}{\tau_o(o)}\kappa^2\beta c = \frac{T_O c}{\pi}(\kappa_p^2 - \beta_p^2)^{\frac{3}{2}} = \frac{r_1 r_2}{r_2-r_1}(\beta_1^2-\beta_2^2) = \frac{a}{2}(3-\sqrt{1+8\tau_{Wo}(O_o)^2}) =$
$= \frac{r_o(o)}{2(2a-r_o(o))}(4a-r_o(o)-\sqrt{(4a-r_o(o))^2-8a(2a-r_o(o))(1-\tau_{Wo}(o)^2)}) = \frac{T_O}{\pi}(\sqrt{1-(\frac{1}{1+z_b})^2})^3 c$
(Schwarzschild radius -system scale)
$a = \frac{R_s}{\kappa^2} = \frac{R_s}{4W} = \frac{\beta_o c}{\omega} \cdot \frac{\sqrt{1-e^2}}{\sqrt{1+e^2+2e\cos(o)}} = \frac{\Delta_{to}(o)}{\tau_o}\beta c$ (semi-major axis)
$m_0 = \frac{\kappa^2 c^2 a}{2G} = 4\pi\rho a^3$ (mass parameter)
$t = \frac{a}{c}$ (temporal scale of the system)
$W = \frac{\beta^2}{2} = \frac{1}{2}(\kappa_o^2 - \beta_o^2) = \frac{1}{4}\kappa_p^2\ (1-e) = \frac{1}{2}(\kappa_o(o)^2 - \frac{\kappa_o(o)^2}{2\delta_o(o)}) = \frac{1}{2}((1-(1+z_{ko}(o))^{-2}) - (1-$
$(1+z_{bo}(o))^{-2}))$ (energy invariant - binding energy)
$\Delta\phi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2} = \frac{2\pi Q^2}{1-e^2} = 6\pi\beta_{int}^2$ (precession of perihelion per orbit)
$h_W = a \cdot \beta c \cdot e_Y$ (angular momentum)
$\omega = \frac{\beta c}{a}$ (angular frequency)
$T = \frac{2\pi}{\omega}$ (orbital period)
$K_{illingV} = \frac{\kappa_{Xp}^2}{\sqrt{1-3\beta_{int}^2}}$ (Killing vector UNTESTED)

Eccentricity Relations

$e = \frac{1}{\delta} - 1 = 1 - \frac{2\beta_a^2}{\kappa_a^2} = \frac{2\beta_p^2}{\kappa_p^2} - 1$ (eccentricity derived from closure)
$e_Y = \sqrt{1-e^2}$ (eccentricity's orthogonal value)
$e_X = \frac{1+e}{1-e} = \frac{r_a}{r_p} = \frac{\delta_a}{\delta_p} = \frac{\beta_p}{\beta_a} = \frac{\kappa_p^2}{\kappa_a^2} = \frac{\kappa_a^2\beta_p^2}{\kappa_p^2\beta_a^2}$ (shape factor)

Time Integration

$\omega_o(o) = \frac{\beta\cdot c}{a} \cdot \frac{(1+e\cdot\cos(o))^2}{(1-e^2)^{\frac{3}{2}}}$ (angular frequency at phase o)
$\Delta_{to}(o) = \int_0^o \frac{1}{\omega_\theta(\theta)}\,d\theta = \frac{a}{\beta c}\tau_o = \frac{T_O}{2\pi}\tau_o$ (time duration of given phase interval)
$\tau_o = (1-e^2)^{\frac{3}{2}} \cdot (\int_0^o (1+e\cdot\cos(\theta))^{-2} d\theta) = \frac{1}{\sqrt{1-e^2}}\int_0^o (\frac{t_o(\theta)}{t})^2 d\theta$ (Temporal Phase interval)

Perihelion Relations

$r_p = a(1-e) = \frac{R_s}{\kappa_p^2}$ (radius at perihelion)
$\kappa_p = \kappa\sqrt{\frac{1}{1-e}} = Q_p\sqrt{\frac{2}{3+e}}$ (potential at perihelion)
$\kappa_{Xp} = \sqrt{1-\kappa_p^2}$ (potential phase projection at perihelion) $\beta_p = \frac{V_p}{c} = \frac{\kappa_p}{\delta\sqrt{2}} = \sqrt{\kappa_p^2 \cdot \frac{1+e}{2}}$
(kinetic at perihelion)
$\delta_p = \frac{\kappa_p^2}{2\beta_p^2} = \frac{1}{1+e}$ (closure factor at perihelion)

 DOI: 10.5281/zenodo.17115270

$Q_p = \sqrt{\kappa_p^2 + \beta_p^2}$ (relational shift at perihelion)

Aphelion Relations

$r_a = a(1+e) = \frac{R_s}{\kappa_a^2}$ (radius at aphelion)
$\beta_a = \sqrt{\beta_p^2 e_X^{-2}} = \beta\sqrt{e_X^{-1}}$ (kinetic projection at aphelion)
$\kappa_a = \sqrt{2W + \beta_a^2}$ (potential projection at aphelion)
$\delta_\mathrm{a} = \frac{1}{1-e} = \frac{\kappa_a^2}{2\beta_a^2}$ (closure factor at aphelion)
$Q_a = \sqrt{\kappa_a^2 + \beta_a^2}$ (relational shift at aphelion)

Phase Variables (Depend on o)

$o =$ orbital phase in radians
$r = r_o(o) = a\frac{1-e^2}{1+e\cos o} = \frac{R_s}{\kappa_o^2}$ (radial distance at phase o)
$\kappa_o = \sqrt{\frac{R_s}{r}} = 1-(1+z_{ko}(o))^{-2} = \kappa_p\sqrt{\frac{1+e\cos(o)}{1+e}}$ (local potential at phase o)
$\kappa_{Xo} = \sqrt{1-\kappa_o^2} = (z_{ko}(o)+1)^{-1}$ (gravitational phase factor at phase o)
$\beta_o(o) = \sqrt{\kappa_o^2 - 2W} = \frac{\kappa_o(o)}{\sqrt{2\delta_o(o)}} = \beta\cdot\frac{\sqrt{1+e^2+2\cdot e\cdot\cos(o)}}{\sqrt{1-e^2}} = \sqrt{\frac{\kappa_o(o)^2}{2}\frac{1+e^2+2\cdot e\cdot\cos(o)}{1+e\cdot\cos(o)}} = \sqrt{\frac{R_s}{r_o(o)} - \frac{R_s}{2a}} = \sqrt{1-(1+z_{bo}(o))^{-2}}$ (kinetic projection at phase o)
$\beta_R(o) = \beta\ \frac{e\sin(o)}{\sqrt{1-e^2}} = \sqrt{\beta_o(o)^2 - \beta_T(o)^2} = \sqrt{((1-(1+z_{ko}(o))^{-2}) - 2W) - \frac{r_o(o)^2\omega_o(o)^2}{c^2}}$ (radial kinetic projection at phase o)
$\beta_T(o) = \frac{r_o(o)\omega_o(o)}{c} = \beta\frac{1+e\cos(o)}{\sqrt{1-e^2}}$ (transverse kinetic projection at phase o)
$\beta_{Yo} = \sqrt{1-\beta_o^2} = (z_{bo}(o)+1)^{-1}$ (relativistic phase factor at phase o)
$\delta_o = \frac{1+e\cos o}{1+e^2+2e\cos o} = \frac{\kappa_o^2}{2\beta_o(o)^2}$ (local closure factor at phase o)
$Q_o = \sqrt{\kappa_o^2 + \beta_o(o)^2}$ (local relational shift vector at phase o)
$\omega_o = a\beta c\frac{e_Y}{r_o^2} = \frac{\beta c}{a}\frac{(1+e\cos o)^2}{(1-e^2)^{3/2}}$ (angular speed at phase o)
$\eta_o = \frac{r}{a} = 2 - \frac{2\beta_o(o)(o)^2}{\kappa_o(o)^2}$ (phase scale amplitude at phase o)
$\tau_{Wo}(o) = \kappa_{Xo}(o)\cdot\beta_{Yo}(o) = Z_{sys}(o)^{-1}$ (phase spacetime factor at phase o)
$t_o = \frac{r_o(o)}{c}$ (light time scale at phase o)
$\Delta_o = 3\beta_{int}^2 \cdot o$ (precession of perihelion at phase o)

Relational Geometry (WILL)

$\theta_1 = \arccos(\beta)$ (distribution angle on S^1, non-physical)
$\theta_2 = \arcsin(\kappa)$ (distribution angle on S^2, non-physical)

$\Delta_Q = Q_o^2 - Q^2$ (phase relational shift amplitude at phase o)
$O_o = \arccos(1-\delta^{-1}) = \arccos(-e) = \arccos(\frac{2\beta_a^2}{\kappa_a^2}-1)$ (orbital balance point where $\kappa_o^2 = 2\beta_o^2$ is true)

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Observer dependant

$Z_{raw}(o) = (1 + \beta_{int}(\cos(o + \omega_i) + e\cos(\omega_i))\sin(i))\ Z_{sys}(o)$ (raw light shift including the line of site Doppler at phase o)
$\beta_{los}(o) = \frac{\beta}{\sqrt{1-e^2}}(\cos(o + \omega_i) + e\cos(\omega_i))\sin(i)$ (line of site light shift)
$i =$ (orbital inclination in relation to line of site and orbital plane)
$\omega_i =$ (phase turn or argument of periapsis)
$K_i = \frac{\beta}{\sqrt{1-e^2}}\sin(i)$ (semi-amplitude invariant)
$Z_{rawmax} = Z_{sys}(o_{i1})(1 + K_i(1 + e\cos\omega_i))$
$Z_{rawmin} = Z_{sys}(o_{i2})(1 + K_i(-1 + e\cos\omega_i))$
$O_{oi} = (O_o + \omega_i)$
$o_{i1} = -\omega_i$
$o_{i2} = \pi - \omega_i$
$Z_{sysi1}(o_{i1}) = (1 - \beta^2\frac{1+e^2+2e\cos(\omega_i)}{1-e^2})^{-\frac{1}{2}}(1 - 2\beta^2\frac{1+e\cos(\omega_i)}{1-e^2})^{-\frac{1}{2}}$
$Z_{sysi2}(o_{i2}) = (1 - \beta^2\frac{1+e^2-2e\cos(\omega_i)}{1-e^2})^{-\frac{1}{2}}(1 - 2\beta^2\frac{1-e\cos(\omega_i)}{1-e^2})^{-\frac{1}{2}}$

Un-tilted 2D coordinates within the orbital plane (UNTESTED)

$x_{orb} = r(o)\cos(o + \omega_i)$
$y_{orb} = r(o)\sin(o + \omega_i)$

Parametric equations for the observed orbit, entirely free of the absolute ICRF background (UNTESTED)

$$\begin{bmatrix} x_{sky} \\ y_{sky} \\ z_{depth} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(i) & -\sin(i) \\ 0 & \sin(i) & \cos(i) \end{bmatrix} \begin{bmatrix} x_{orb} \\ y_{orb} \\ 0 \end{bmatrix}$$

$x_{sky} = r(o)\cos(o + \omega_i)$
$y_{sky} = r(o)\sin(o + \omega_i)\cos(i)$
$z_{depth} = r(o)\sin(o + \omega_i)\sin(i)$

Beautiful and Interesting Connections

$\frac{r_a}{r_p} = \frac{1+e}{1-e} = \frac{\beta_p}{\beta_a} = \frac{\kappa_p^2}{\kappa_a^2} = \frac{\kappa_a^2\beta_p^2}{\kappa_p^2\beta_a^2}$ (remarkable equivalence of structural (κ on S^2 carrier) and dynamical (β on S^1 carrier) asymmetry confirming once again that $SPACETIME \equiv ENERGY$)

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