

Derivation of Light Deflection *Within WILL Relational Geometry*

Derived from WILL Part I: Relational Geometry
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Abstract

We derive the gravitational deflection of light within the WILL Relational Geometry (RG) framework, tracing every step to its foundational dependencies. The derivation proceeds from the Single-Axis Transformation Principle (for photons, $\beta = 1 \Rightarrow \beta_Y = 0$), which doubles the effective gravitational potential relative to massive particles, through the field equation $\kappa^2 = R_s/r$ and the transverse impulse integral. The result,

$$\alpha = 2\kappa_0^2 = \frac{2R_s}{r_0} = \frac{4GM}{c^2 r_0},$$

reproduces the General Relativistic prediction with no metrics, tensors, or free parameters — relying solely on the closure properties of the S^1 and S^2 relational carriers.

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1 Dependency Map

The light deflection formula rests on the following chain of WILL RG results. Each numbered item references the theorem or definition in WILL Part I from which it derives.

Dependency Chain

- D1. Ontological Principle:** SPACETIME \equiv ENERGY (Principle 2.6)
- D2. Relational Carriers:** S^1 (1 DOF, kinematic) and S^2 (2 DOF, gravitational) are the unique minimal carriers (Theorem 2.13)
- D3. Carrier Closure:** $\beta^2 + \beta_Y^2 = 1$ on S^1 ; $\kappa^2 + \kappa_X^2 = 1$ on S^2
- D4. Energetic Closure Condition:** $\kappa^2 = 2\beta^2$ (Theorem 6.5, from DOF ratio $\mathcal{R} = 2$)
- D5. Energy-Symmetry Law:** $W_{\text{mass}} = \frac{1}{2}(\kappa^2 - \beta^2)$ (dual-axis partition for massive particles)
- D6. Single-Axis Transformation Principle:** For light, $\beta = 1$, $\beta_Y = 0 \Rightarrow W_\gamma = \kappa^2 - \beta^2$ (no $\frac{1}{2}$ factor)
- D7. Field Equation:** $\kappa^2 = R_s/r$ (unified field equation)
- D8. Gravitational Potentials:** $\Phi_{\text{mass}} = \frac{1}{2}\kappa^2 c^2$; $\Phi_\gamma = \kappa^2 c^2$ (from D5 and D6)
- D9. Light Deflection:** $\alpha = 2\kappa_0^2 = 2R_s/r_0$ (derived below)

2 Prerequisites from WILL Part I

We collect the essential results needed for the derivation.

2.1 Gravitational Potential for Massive Particles

For a massive particle, the Energy-Symmetry Law partitions the transformation resource equally between the two orthogonal axes of S^1 (Phase β_Y and Amplitude β). The specific energy invariant for orbital motion is:

$$W_{\text{mass}} = \frac{1}{2}(\kappa^2 - \beta^2), \quad (1)$$

where the factor $\frac{1}{2}$ arises from the dual-axis distribution. This invariant is conserved for closed systems and gives rise to the effective gravitational potential:

$$\Phi_{\text{mass}} = \frac{1}{2}\kappa^2 c^2 = \frac{R_s c^2}{2r}. \quad (2)$$

2.2 Single-Axis Transformation Principle (Light)

For light, the kinematic projection saturates the carrier:

$$\beta = 1 \implies \beta_Y = \sqrt{1 - \beta^2} = 0. \quad (3)$$

This has three critical consequences:

1. The Phase component (β_Y , Y-axis) vanishes entirely — no internal state evolution exists.
2. No rest frame exists for self-centering, eliminating the dual-framing that justifies the $\frac{1}{2}$ partitioning.
3. The entire transformation resource concentrates on the single X-axis (Amplitude component).

Consequently, the energy invariant for photon interactions with a gravitating body is:

$$W_\gamma = \kappa^2 - \beta^2 = \kappa^2 - 1 \quad (\text{no } \frac{1}{2} \text{ factor}). \quad (4)$$

The effective gravitational potential for light is therefore:

$$\boxed{\Phi_\gamma = \kappa^2 c^2 = \frac{R_s c^2}{r} = 2 \Phi_{\text{mass}}.} \quad (5)$$

Remark 2.1 (Origin of the Factor of 2). *The ratio $\Phi_\gamma/\Phi_{\text{mass}} = 2$ is not an empirical fitting parameter. It is a topological consequence of the S^1 carrier architecture: massive particles distribute their relational resource across two orthogonal axes (β, β_Y), while light concentrates it on one. The factor of 2 is the same 2 that appears in the closure condition $\kappa^2 = 2\beta^2$ — the DOF ratio of S^2 to S^1 .*

Remark 2.2 (Consistency Check at Zero Gravity). *At $\kappa = 0$ (no gravitational interaction):*

$$\begin{aligned} W_{\text{mass}} &= \frac{1}{2}(0 - 0) = 0, \\ W_\gamma &= 0 - 1 = -1. \end{aligned}$$

The value $W_\gamma = -1$ equals the full rest-energy cost of the transition to a photon state. The partitioned form would give $-\frac{1}{2}$, violating energy symmetry.

3 Derivation of Light Deflection

3.1 Setup: Photon Flyby Geometry

Consider a photon traveling from spatial infinity, passing a gravitating body (characterized by Schwarzschild radius R_s) at closest approach distance r_0 , and continuing to spatial infinity. In the weak-field regime ($\kappa_0^2 = R_s/r_0 \ll 1$), the unperturbed trajectory is a straight line.

Definition 3.1 (Impact Parameter). *The impact parameter b is the perpendicular distance from the gravitating center to the unperturbed (straight-line) photon path. In the weak-field limit, $b \approx r_0$ to leading order.*

We parametrize the photon's unperturbed path along the x -axis at speed c :

$$x(t) = ct, \quad y = b = \text{const}, \quad (6)$$

so the instantaneous distance to the gravitating center is:

$$r(t) = \sqrt{b^2 + c^2 t^2}. \quad (7)$$

3.2 Gravitational Acceleration from the WILL RG Potential

From equation (5), the effective gravitational potential for a photon is:

$$\Phi_\gamma(r) = \kappa^2 c^2 = \frac{R_s c^2}{r}. \quad (8)$$

The magnitude of the gravitational acceleration experienced by the photon is:

$$g(r) = \left| \frac{d\Phi_\gamma}{dr} \right| = \frac{R_s c^2}{r^2}. \quad (9)$$

Remark 3.2 (Comparison with Massive Particles). *For a massive particle, $g_{mass}(r) = |d\Phi_{mass}/dr| = R_s c^2 / (2r^2)$, which is exactly half of equation (9). This factor-of-2 difference is the geometric signature of axis count in relational space.*

3.3 Transverse Impulse Integral

The component of gravitational acceleration *perpendicular* to the photon's unperturbed path is:

$$g_\perp(t) = g(r(t)) \cdot \frac{b}{r(t)} = \frac{R_s c^2 b}{(b^2 + c^2 t^2)^{3/2}}. \quad (10)$$

The total transverse velocity impulse accumulated over the entire flyby is:

$$\Delta v_\perp = \int_{-\infty}^{+\infty} g_\perp(t) dt = R_s c^2 b \int_{-\infty}^{+\infty} \frac{dt}{(b^2 + c^2 t^2)^{3/2}}. \quad (11)$$

Lemma 3.3 (Evaluation of the Standard Integral).

$$\int_{-\infty}^{+\infty} \frac{dt}{(b^2 + c^2 t^2)^{3/2}} = \frac{2}{b^2 c}. \quad (12)$$

Proof. Substitute $u = ct/b$, so $dt = (b/c) du$ and $b^2 + c^2 t^2 = b^2(1 + u^2)$:

$$\begin{aligned} \int_{-\infty}^{+\infty} \frac{dt}{(b^2 + c^2 t^2)^{3/2}} &= \frac{1}{b^3} \int_{-\infty}^{+\infty} \frac{b/c du}{(1 + u^2)^{3/2}} \\ &= \frac{1}{b^2 c} \int_{-\infty}^{+\infty} \frac{du}{(1 + u^2)^{3/2}}. \end{aligned} \quad (13)$$

The remaining integral evaluates to 2 (by the substitution $u = \tan \phi$):

$$\int_{-\infty}^{+\infty} \frac{du}{(1 + u^2)^{3/2}} = \left[\frac{u}{\sqrt{1 + u^2}} \right]_{-\infty}^{+\infty} = 1 - (-1) = 2. \quad \square$$

Substituting (12) into (11):

$$\Delta v_\perp = R_s c^2 b \cdot \frac{2}{b^2 c} = \frac{2R_s c}{b}. \quad (14)$$

3.4 Deflection Angle

The deflection angle is the ratio of the acquired transverse velocity to the longitudinal velocity (which remains c to leading order):

$$\alpha = \frac{\Delta v_{\perp}}{c} = \frac{2R_s}{b}. \quad (15)$$

In the weak-field limit $b \approx r_0$, giving the final result:

Light Deflection in WILL RG

$$\alpha_{\text{WILL}} = 2\kappa_0^2 = \frac{2R_s}{r_0} = \frac{4GM}{c^2 r_0} \quad (16)$$

where:

- $\kappa_0 = \sqrt{R_s/r_0}$ is the potential projection at closest approach (S^2 carrier),
- $R_s = 2GM/c^2$ is the Schwarzschild radius (system scale),
- r_0 is the closest approach distance (impact parameter).

4 Anatomy of the Result

4.1 Why the Factor of 2 (and Why It Appears Twice)

It is instructive to decompose the deflection into its physical origin within the WILL RG framework:

Entity	Potential	Deflection at $v = c$
Massive particle	$\Phi_{\text{mass}} = \frac{1}{2}\kappa^2 c^2$	$\alpha_{\text{mass}} = \frac{R_s}{b\beta_{\infty}^2}$
Massive at $\beta = 1$	$\frac{1}{2}\kappa^2 c^2$	$\frac{R_s}{b}$
Photon (WILL)	$\Phi_{\gamma} = \kappa^2 c^2$	$\frac{2R_s}{b}$

In General Relativity, the factor of 2 over the ‘‘Newtonian’’ prediction arises from the sum of time-time (g_{00}) and space-space (g_{rr}) metric contributions. In WILL RG, both contributions are unified into a single geometric principle: the collapse of $\beta_Y \rightarrow 0$ removes the dual-axis energy partitioning, doubling the effective gravitational coupling.

4.2 Expression in Pure WILL RG Variables (No G , No Mass)

The deflection angle can be expressed entirely in terms of operationally measurable relational projections, without reference to G or mass:

$$\alpha = 2\kappa_0^2, \quad (17)$$

where κ_0 is determined from any of:

- **Direct spectroscopy:** $\kappa_0^2 = 1 - (1 + z_k)^{-2}$ from gravitational redshift at r_0 ,

- **Orbital data:** $\kappa_0^2 = R_s/r_0$ with $R_s = \kappa^2 a$ from any orbiting body,
- **Two-point method:** $R_s = \frac{r_1 r_2}{r_2 - r_1} (\beta_1^2 - \beta_2^2)$,
- **Classical calibration:** $R_s = 2GM/c^2$ (treating G and M as output bookkeeping).

4.3 Connection to Precession and Photon Sphere

The light deflection formula connects naturally to other WILL RG results:

1. **Precession formula** (bound orbits): $\Delta\varphi = \frac{2\pi Q^2}{1-e^2} = \frac{3\pi R_s}{a(1-e^2)}$.

For massive particles, the total relational shift is $Q^2 = \beta^2 + \kappa^2 = 3\beta^2$ under closure. For light, the single-axis principle enhances the gravitational contribution: $Q_\gamma^2 = \beta^2 + 2\kappa^2 = 1 + 2\kappa^2$.

2. **Photon sphere:** At the causal dynamic horizon ($Q = 1$, $\kappa^2 = 2/3$, $\beta^2 = 1/3$), the precession reaches $\Delta\varphi = 2\pi$ — a full topological closure. This is the regime where light deflection transitions from perturbative ($\alpha \ll 1$) to non-perturbative (closed null orbits).
3. **Regime correspondence:**
 - Weak field ($\kappa_0^2 \ll 1$): $\alpha \approx 2\kappa_0^2$ (this derivation),
 - Photon sphere ($\kappa_0^2 = 2/3$): $\Delta\varphi = 2\pi$ (topological closure),
 - Strong lensing ($\kappa_0^2 \sim 1$): requires the full non-linear trajectory integration.

5 Numerical Verification: Sun-Grazing Photon

Parameter	Value
Solar mass M_\odot	1.989×10^{30} kg
Solar radius R_\odot	6.957×10^8 m
Gravitational constant G	6.674×10^{-11} m ³ kg ⁻¹ s ⁻²
Speed of light c	2.998×10^8 m/s
$R_s(\text{Sun})$	2953.8 m
$\kappa_0^2 = R_s/R_\odot$	4.246×10^{-6}
κ_0	2.061×10^{-3}

Computing the deflection:

$$\begin{aligned}
\alpha &= 2\kappa_0^2 = 2 \times 4.246 \times 10^{-6} = 8.492 \times 10^{-6} \text{ rad} \\
&= 8.492 \times 10^{-6} \times 206\,265 = \mathbf{1.751} \text{ arcseconds.}
\end{aligned}
\tag{18}$$

Source	Deflection (arcsec)
WILL RG prediction	1.751
GR prediction	1.7512
Eddington 1919 measurement	1.75 ± 0.06
Modern VLBI	1.7520 ± 0.0002

6 Summary of the Logical Chain

Complete Derivation Path

1. SPACETIME \equiv ENERGY (Ontological Principle)
2. $\Rightarrow S^1, S^2$ as unique minimal relational carriers
3. $\Rightarrow \beta^2 + \beta_Y^2 = 1; \quad \kappa^2 + \kappa_X^2 = 1$ (carrier closure)
4. \Rightarrow Energy-Symmetry Law: $W_{\text{mass}} = \frac{1}{2}(\kappa^2 - \beta^2)$
5. \Rightarrow For light ($\beta = 1, \beta_Y = 0$): $W_\gamma = \kappa^2 - 1$ (single-axis, no $\frac{1}{2}$)
6. \Rightarrow Effective potential: $\Phi_\gamma = \kappa^2 c^2 = 2\Phi_{\text{mass}}$
7. \Rightarrow Field equation: $\kappa^2 = R_s/r$ gives $g_\gamma(r) = R_s c^2 / r^2$
8. \Rightarrow Transverse impulse: $\Delta v_\perp = \int g_\perp dt = 2R_s c / r_0$
9. \Rightarrow $\boxed{\alpha = 2\kappa_0^2 = 2R_s / r_0 = 4GM / (c^2 r_0)}$

What This Derivation Does NOT Use

- No spacetime metric ($g_{\mu\nu}$)
- No Einstein field equations or Christoffel symbols
- No geodesic equation
- No weak-field expansion of the Schwarzschild metric
- No free parameters

The entire result follows from: (i) the closure architecture of S^1 and S^2 , (ii) the single-axis principle for $\beta = 1$, and (iii) the field equation $\kappa^2 = R_s/r$.