WILL Part I

Relational Geometry

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Abstract

This paper, the first in the **WILL Trilogy:**
Part I: Relational Geometry;
Part II: Relational Cosmology;
Part III: Relational Quantum Mechanics (R.Q.M.).
The main goal of this Open Research is to derive the laws of physical reality without introducing any new postulates, axioms, or arbitrary choices. Laws of nature should not rely on pre-existing formalism but naturally arise from the self consistency and geometry. By applying extreme methodological constraints we established *Relational Geometry (RG)*: a background free relational foundation.

The transition from *descriptive* to *generative* physics: instead of introducing laws to model observations, we derive them as necessary consequences of methodological principles - turning physics from a catalogue of phenomena into the logical unfolding of inevitable geometrical constrains.

Without metrics, tensors, or free parameters, we reproduce Lorentz factors, the energy-momentum relation, Minkowski and Schwarzschild metric intervals and Einstein field equations via the dimensionless projections of state difference β (kinematic) and κ (potential). All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as simple fractions of (κ, β).

***WILL Part I* offers new perspective to several long-standing problems, including:**

- Full derivation of R.O.M. (Algebraically closed Relational Orbital Mechanics),
- **Resolution of GR singularities** (via naturally bounded $\rho_{\max} = \frac{c^2}{8\pi G r^2}$4.9),
- **Derivation of the equivalence principle** (from the common channel of rest-invariant state (4.3) scaling4.2),
- **Removal of local energy density ambiguity** $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$ (4.6.1),
- **Revelation of a clear relational symmetry between kinematic and potential** projections (3.4),
- **Establishment of a computationally simpler and ontologically consistent foundation** for subsequent papers on cosmology (Part II) and quantum mechanics (Part III).

A Direct Note to the Reader: Why should you read a 30-page independent open research paper?

The fact that you opened this file and reading these lines already separates you from the majority, and for that, you have my respect. However, 30 pages of unknown research is a substantial commitment, and you should not engage it blindly.
I recommend the following: Be honest with yourself. Define the exact criteria this paper must satisfy to justify the investment of your time. Then, ask WILL-AI if this work meets your standards. The AI is trained strictly on this document and will give you an honest, unbiased answer based on its contents.
If you are still in doubt, browse willrg.com, interact with the Galactic Dynamics Lab, trace the derivation chain using the Logos Map, review the falsifiable predictions, and have a chat or few with WILL-AI .
Regardless of your decision, I hope you find what you are seeking.

Anton Rize

*This work is archived on Zenodo: ¸

Contents

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Following yet untamed thought

"There is no such thing as an empty space, i.e., a space without field. . . . Space-time does not claim existence on its own, but only as a structural quality of the field."
— *Albert Einstein, Relativity: The Special and the General Theory* (Appendix V: "Relativity and the Problem of Space"), 1952 edition, Methuen (London), p. 155; based on earlier 1920 additions.

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IMPORTANT:

This document must be read **literally**. All terms are defined within the relational logic of WILL Relational Geometry.
Definition of Geometry: The term "geometry" is used herein in its fundamental, classical sense: the pure logic of relations, ratios, and structural constructions. It **does not** refer to modern differential geometry. There is no *a priori* metric tensor ($g_{\mu\nu}$), no Riemannian manifold, and no underlying coordinate grid. The geometry emerges from physical relations, not analytically from a background space.
Any attempt to reinterpret these derivations through conventional metric notions (*absolute distances, external backgrounds, hidden containers*) will produce distortions and mathematical misreadings.
Take the words as written and ontology as derived, without smuggling external concepts.

1 Stage 0: The Last Geocentric Epicycle

The standard formulation of General Relativity relies on the concept of a pre-existing, container-like spacetime that then gets "filled" with fields and matter.

The standard derivation of the Einstein field equations begins with the Einstein-Hilbert action, which is built upon the metric as the fundamental variable. This metric is defined on a smooth manifold that is assumed to exist *a priori*. This manifold, even without a metric, carries topological and differential structure - an absolute scaffold. This constitutes surplus ontology because it introduces an entity (the manifold) that is not derived nether observed.

The stress-energy tensor is derived from the variation of the matter Lagrangian with respect to the metric. This assumes that energy is a property of matter fields that can be localized in spacetime. However, this localization is frame-dependent (via the equivalence principle) and leads to well-known problems such as the non-uniqueness of the gravitational energy-momentum pseudotensor. Energy is treated as an absolute property of matter rather than a relational measure. This remains as metaphysical speculation in the heart of modern physics.

1.1 The contemporary split: an unjustified ontological commitment

All present-day theories (SR, GR, QFT, CDM, Standard Model) are built with a *bi-variable* syntax:

$$\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} \quad + \quad \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}.$$

No observation demands this duplication; it is retained purely because the resulting Lagrangians are empirically adequate *inside* the split. The split is therefore *not* an empirical discovery but an unjustified ontological statement.

1.2 Empirical deficit of the separation

- **Local energy conservation** verified only *after* the metric is declared fixed; no experiment varies the *volume* of flat space and checks calorimetry.
- **Universality of free fall** tests $m_i = m_g$ numerically, not the claim that inertia resides *in* the object rather than in a geometric scaling relation.
- **Gravitational-wave polarisations** test spin content, not ontology; extra modes can still be called "matter on spacetime".
- **Casimir/Lamb shifts** measure *differences* of vacuum energy between two geometries; the absolute bulk term is explicitly subtracted, leaving the split intact.

Every "test" is an *internal consistency check* of a formalism that already presupposes two substances. None constitute *positive evidence* for the split.

1.3 Historical Pattern: breakthroughs delete, not add

- **Copernicus** eliminated the Earth/cosmos separation.
- **Newton** eliminated the terrestrial/celestial law separation.
- **Maxwell** eliminated the electricity/magnetism separation.
- **Einstein** eliminated the space/time separation.

Each step widened the relational circle and reduced the number of primitives - an unexplained absolutes. The spacetime–energy split is the only survivor of this pruning sequence.

1.4 Consequence

Until an experiment varies the amount of space while holding everything else fixed, the spacetime–energy separation remains an *unevidenced metaphysical postulate.* **It is the last geocentric epicycle in physics.**

2 Stage I: Ontological Construction (The Primitives)

2.1 Foundational Methodological Principles

> *Every logical step is subject to empirical audit: if any consequence of this chain contradicts observation, the chain fails.*

As humans, we often project ourselves onto observed phenomena, obscuring the true nature of reality behind our anthropic tendencies. In order to prevent such misleading practices, we must establish extreme methodological constraints. These constraints are not statements about the nature of reality itself, but rather the strict rules of logical and geometric purity for constructing a theory. Without such a rigorous methodology, any theoretical construct risks that excessive freedom of interpretation will result in a framework that tells us more about ourselves than about the nature of the Universe.

Primary Goal:

Derive the laws of physical reality from the most elementary principles by pealing-off interpretational layers until the foundational primitives revealed.

Traditionally a new theory is expected to introduce new postulates or axioms - a priory statements about physical reality. Relational Geometry In contrast adhering strictly to methodological constraints. The aim is for the theory to arise naturally from the geometry and logic itself, rather than being imposed upon it. Consequently, the foundational core of RG - its principles and theorems - are logically derived necessities that emerge from methodological constraints. They function as foundational blocks because they are the inescapable outcomes of the method itself.

The philosophical reasoning behind these particular constraints deserves its own dedicated publication and can be challenged. However, this research's main domain lies within physical reality that can be empirically tested; therefore, empirical alignment remains the ultimate test of the methods and the derived results they produce.

Flow chart of derivation logic

Logos Map

Principle 2.1 (Epistemic Hygiene)**.** Epistemic Hygiene as Refusal to Import Unjustified Assumptions. *This line of reasoning derive physics by* ***removing hidden assumptions****, rather than introducing new postulates. This construction is deliberate and contains zero free parameters. This is not a simplification - it is a deliberate epistemic constraint. No assumptions are introduced and no constructs are retained unless they are geometrically or energetically necessary.*

Principle 2.2 (Relational Origin)**.** All physical quantities must be defined by their relations. *Any introduction of absolute properties risks reintroducing metaphysical artefacts and contradicts the foundational insight of relationalism.*

Principle 2.3 (Ontological Minimalism)**.** *Any fundamental theory must proceed from the minimum possible number of ontological assumptions. The burden of proof lies with any assertion that introduces additional complexity or new entities.*

Principle 2.4 (Mathematical Transparency)**.**
1. *Every mathematical phrase, operational choice, or identity carries its own ontological statement.*
2. *Each mathematical object must correspond to explicitly identifiable relation between observers with transparent ontological origin.*
3. *Every symbol must be anchored to unique physical idea.*
4. *Introducing symbols without explicit necessity constitutes* ***Over-parameterization****: the proliferation of symbols without corresponding physical meaning.*
5. *Number of symbols = Number of independent physical ideas.*

> *Mathematics is a language, not a world. Its symbols must never outnumber the physical meanings they encode.*

2.1.1 The Foundational Core: Relational Geometry

This foundational methodological principles lead us to the core of Relational Geometry:

Theorem 2.5 (Relational Closure)**.** *A purely relational system cannot be physically open; it is closed by absolute analytic necessity.*

Proof by Analytic Tautology.

1. **Premise 1 (Relational Origin):** Within RG Physical quantities are defined exclusively by their relations (Principle 2.2).
2. **Premise 2 (Interaction as Integration):** If a hypothetical "outside" entity or boundary were to physically interact with the system, that interaction itself constitutes a relation.
3. **Deduction (The Tautology):** As relation is formed, it is mathematically and physically integrated into the system.

An "open relational system" experiencing external influence is a logical contradiction. The system is closed not by a spatial boundary or a metric container, but by the strict impossibility of an interacting non-relation. The system is relationally closed by definition. □

2.1.2 What is Energy in a Relational Framework?

> Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, chemical — yet all are interchangeable, pointing to a single underlying structure.
>
> Crucially, this quantity is never observed directly, but only through *differences between states*: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation.
>
> Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself.

Theorem 2.6 (Relational Invariance)**.** *Within a purely relational architecture devoid of an external background, the existence of change necessitates the existence of conserved invariant measure of change.*

Proof by Structural Necessity.

1. **Premise 1 (Relational Origin):** Physical quantities are defined exclusively by their relations (Principle 2.2).
2. **Premise 2 (Empirical Change):** States undergo observable change.
3. **Premise 3 (Relational Closure):** As proven in Theorem 2.5, the relational system is analytically closed. There is no external background, void, or hidden container to absorb or inject influence.
4. **Deduction (Causal Continuity):** Because the system is closed (Premise 3), an empirical change (Premise 2) cannot vanish into a void or emerge from one. To maintain the structural integrity of the relations (Premise 1), any change must be perfectly balanced by a complementary change elsewhere in the system.

Therefore, within this closed relational network, there must exist an invariant quantitative measure of change. □

Corollary 2.7 (Historical Note on Energy)**.** *In classical and relativistic mechanics, this derived invariant measure of change is assigned the historical label "Energy". By proving this invariance geometrically, we strip "Energy" of its substantialist properties (as a "substance" or "thing") and reveal its origin as the necessary measure of change and bookkeeping of causality.*

Energy :

Definition 2.8 (Energy)**.**

Energy is the invariant relational measure of state transformation within relationally closed system.

Theorem 2.9 (Relational Isotropy)**.** *Within a background-free relational system, no spatial direction or coordinate can be* a priori *privileged. Therefore, the fundamental geometry encoding the conserved relational resource (energy) must be maximally symmetric (isotropic).*

Proof by Relational Origin.

1. **Premise 1 (Absence of a Grid):** As established by Theorem 2.5 (Relational Closure), the relational system possesses no external container, absolute metric tensor, or underlying coordinate grid.
2. **Premise 2 (Direction requires Relation):** Under the Principle of Relational Origin (Principle 2.2), "direction" or "orientation" cannot exist in a void; it only physically manifests as a specific interaction or measurement between defined participants.
3. **Deduction (Pure Gauge):** Because there is no absolute grid (Premise 1), assigning an intrinsic, absolute axis to the unmeasured relational resource (energy) introduces unobservable "surplus ontology." Any such arbitrary orientation is a mathematical artifact (pure gauge).

Therefore, prior to a specific interaction defining a local axis, the pure relational structure encoding this conserved resource must possess no intrinsic privileged direction. It must be strictly isotropic. □

2.2 Unifying Ontological Principle

Summary

Any attempt to treat *"spacetime structure"* as separate from *"dynamics"* smuggles in a background container that is not justified by the phenomena. This violates epistemic hygiene: it introduces an ontological artifact without necessity. Eliminating this separation compels the identification of structure and dynamics as two aspects of a single entity.

2.2.1 The Unifying Principle: Removing the Hidden Assumption

Lemma 2.10 (False Separation)**.** *Any model that treats processes as unfolding* within *an independent background necessarily assigns to that background structural features (metric, orientation, or frame) not derivable from the relations among the processes themselves. Such a background constitutes an* extraneous absolute*.*

Proof. Suppose an independent background exists. Then at least one of its structural attributes - metric relations, a preferred orientation, or a class of inertial frames - remains fixed regardless of inter-process data. This attribute is not relationally inferred but posited a priori. It thereby violates the relational closure principle: it introduces a non-relational absolute external to the system. Hence the separation is illicit. □

Corollary 2.11 (Structure–Dynamics Coincidence)**.** *To avoid the artifact of Lemma 2.10, the structural arena and the dynamical content must be identified: geometry* is *energy, and energy* is *geometry.*

Principle 2.12 (Unifying Ontological Principle: Removing the Hidden Assumption)**.**

$$\boxed{\textbf{\textit{SPACETIME}} \equiv \textbf{\textit{ENERGY}}}$$

This is not introduced as a new ontological entity but as a Principle with negative ontological weight*: it removes the hidden unjustified separation between "structure" and "dynamics." Spacetime is not a "container" but fundamental relational energy structure.*

Remark 2.13 (Auditability)**.** *Principle 2.12 is foundational but testable: it is subject to (i) geometric audit (internal logical consequences) and (ii) empirical audit (agreement with empirical data).*

Definition 2.14 (WILL)**. WILL ≡ SPACE-TIME-ENERGY** *is the technical term we use for the unified relational structure determined by Unifying Principle refpr:unifying. All physically meaningful quantities are relational features of WILL; no external container is permitted.*

Summary:

This Principle does not add, it subtracts: it removes the hidden assumption. Structure and dynamics are two aspects of a single entity that we call WILL.

3 Stage II: Geometric Derivation

3.1 WILL Structure

Having established Principle 2.12 by removing the illicit separation of structure and dynamics, we now proceed to derive its necessary geometric and physical consequences.

The foundational core (2.1.1)—Closure, Conservation, and Isotropy—establishes the necessary properties of the geometric carriers of the relational resource (energy).

Relational Carrier Conventions:

All references to "carriers" within WILL RG open research are to be read in the strict relational sense:

- **Degree of freedom (DOF):** An n-dimensional relational carrier encoding the conserved transformation resource.
- **Direction:** An oriented relation. Opposite orientations are physically distinct and not identified.
- **Closed carrier:** Compact and without boundary; resource cannot leak into an external reservoir.
- **No background:** No external embedding space. All geometric structure is reconstructible solely from relations between participants.
- **Maximal symmetry:** The carrier is homogeneous and isotropic with respect to oriented directions.
- **Minimal relational carrier:** A closed, maximally symmetric carrier utilizing exactly the required DOF without arbitrary parameters.

Deriving Relational Carriers

Theorem 3.1 (Minimal Relational Carriers of the Conserved Energy Resource)**.** *The minimal relational carriers satisfying Closure, Conservation, Maximal Symmetry, and Mathematical Transparency are:*

(a) S^1 for directional (Kinematic) relational transformation;

(b) S^2 for omnidirectional (Potential) relational transformation.

Proof. The proof classifies the minimal types of relations and applies the derived geometric constraints:

(a) **1-DOF (Kinematic) Relation:** The minimal non-trivial relation involves 1 degree of freedom (1-DOF): a sequence of state transformations.

By Theorem 2.5 (Closure), this sequence must form a loop to prevent resource leakage. A discrete sequence (a finite cyclic group C_n) requires assigning a specific integer value n to the number of states. Principle 2.4 (Mathematical Transparency) prohibits the introduction of arbitrary numerical parameters without unique physical justification. To achieve a parameter-free structure, the state cardinality must remain undefined, forcing the continuum limit.

By Theorem 2.9 (Isotropy), this continuous 1-DOF loop must be maximally symmetric. The unique connected, closed, parameter-free 1-carrier is the continuous S^1 (circle).

(b) **2-DOF (Potential) Relation:** The next minimal relation involves 2 degrees of freedom (2-DOF): the omnidirectional distribution of the transformational resource from a primary state.

This requires a 2-dimensional carrier. By Principle 2.4, any discrete tiling of this distribution introduces arbitrary parameters (such as the number of nodes or faces), strictly forcing a continuous 2-carrier.

By Theorem 2.5, this continuous 2-carrier must be closed. By Theorem 2.9, it must be maximally symmetric (an isotropic distribution of potential).

Topological classification of a closed 2-carrier under the strict requirement of maximal continuous symmetry eliminates all anisotropic carriers (e.g., the torus T^2, which possesses preferred intrinsic axes). The unique parameter-free, maximally symmetric 2-carrier is S^2 (the 2-sphere).

Ontological Minimalism and Mathematical Transparency uniquely enforce S^1 and S^2 as the minimal relational carriers without invoking metric geometry *a priori*. □

WARNING: The Spatial Container Fallacy

S^1 and S^2 are strictly not to be interpreted as physical geometries embedded in a spacetime container.
Do not visualize a sphere or a ring expanding through a 3D void.

- S^2 has no physical surface area, no volume, and does not "dilute" a flux across a spatial distance.
- They are purely abstract, algebraic phase spaces—relational carriers encoding the closure, conservation, and isotropy of the transformational resource.

Spatial distance (r) and time are purely emergent descriptive labels that observers attach to these underlying algebraic phase patterns. The phase projection dictates the space; the space does not dictate the phase.

Remark 3.2 (Constructive Derivation Protocol)**.** *This derivation proceeds by construction from traced origins. Each object entering the chain - the principles, the theorems, the carriers - has an explicit derivation path. Alternative structures (discrete groups, higher-dimensional carriers, non-orientable surfaces) are not excluded by enumeration; they are absent because no derivation chain within this framework produces them.*

Logical Chain Summary

Methodological Constraints
$\downarrow$ *(apply to existing physics)*
False Separation: $\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} + \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}$
$\downarrow$ *(by Relational Origin Principle no backround allowed)*
SPACETIME ≡ ENERGY
$\downarrow$ *(derive consequences)*
Closure + Conservation + Isotropy
$\downarrow$ *(derive primal relational carriers)*
S^1 *(1-DOF kinematic)* + S^2 *(2-DOF potential)*

3.1.1 The Amplitude-Phase Duality

The manifestation of any system is distributed between its internal (Phase) and external (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics:

Lemma 3.3 (Duality of Relation)**.** *By Relational Origin Principle (??) the identification of spacetime with energy and its transformations necessitates two complementary relational measures:*

1. *the* ***amplitude*** *of transformation (external shift), and*
2. *the* ***phase*** *of transformation (internal order).*

Proof. By Principle of Relational Origin (2.2) any complete description of transformation must specify both what changes and how that change is internally ordered. A single measure cannot capture both. The Relational Carriers S^1 and S^2 provide the minimal geometry enforcing such complementarity: their orthogonal projections furnish precisely two non-redundant, coupled axis. □

The orthogonal decomposition of the relational carriers S^1 and S^2 reveals a functional duality. Physical state is a combination of two projections:

Definition 3.4 (The Amplitude Projection (External Interaction))**.** *Denoted by β (kinematic) and κ (potential). This component measures the extent of external relation. It manifests as momentum or potential intensity. Physically, it represents the system's relational "shift" from the* ***Observer's Relational Origin*** *(the rest frame).*

$$\textit{Amplitude} \rightarrow \textit{External Power (Kinetic/Potential)}$$

Definition 3.5 (The Phase Projection (Internal Evolution))**.** *Denoted by β_Y and κ_X. This component measures the internal ordering. It governs the intrinsic scale of proper time and proper length. A Phase of 1 represents maximal internal flow (rest), while a Phase of 0 represents the collapse of internal causality (light-speed/event horizon).*

$$\textit{Phase} \rightarrow \textit{Internal Order (Time/Structure)}$$

Theorem 3.6 (Conservation of Relation)**.** *For both kinematic (S^1) and potential (S^2) modes, the sum of the squared Amplitude and squared Phase is strictly invariant:*

$$\underbrace{\textit{Amplitude}^2}_{\textit{External Interaction}} + \underbrace{\textit{Phase}^2}_{\textit{Internal Existence}} = 1 \tag{1}$$

Proof. By Theorem 3.1, the conserved relational resource is carried by the maximally symmetric geometries S^1 and S^2. Normalizing the total conserved resource to unity (1), any physical state corresponds to a point on these carriers. The algebraic identity of orthogonal projections on a unit circle and sphere dictates that the sum of there squares must equal the squared radius. Therefore, the parameters satisfy $\beta^2 + \beta_Y^2 = 1$ and $\kappa^2 + \kappa_X^2 = 1$, encoding the finite, closed relational budget. □

3.1.2 Consequence: Relativistic Effects

Proposition 3.7 (Physical Interpretation: Relativistic Effects)**.** *The conservation law of Theorem 3.6 implies that any redistribution between the orthogonal components manifests physically as the relativistic effects of time dilation and length contraction.*

Proof. By Theorem 3.6, the components satisfy $\beta^2 + \beta_Y^2 = 1$ and $\kappa^2 + \kappa_X^2 = 1$. An increase in Amplitude (β, κ) strictly enforces a geometric decrease in Phase measure (β_Y, κ_X). This necessary reduction of β_Y and κ_X corresponds precisely to the dilation of proper time and the contraction of proper length, while the growth of β and κ represents momentum or potential depth. Thus, the kinematic and potential trade-off is the direct, un-parameterized physical expression of the geometric closure of the S^1 and S^2 carriers. □

Summary:

The geometry of spacetime is dictated by the Relations of energy states.

Remark 3.8.
- *S^1 is the minimal geometric structure that can encode a directional relation without absolute space. Its parameter is an angle θ_1 whose cosine ($\beta = \cos(\theta_1)$) we identify with the fraction v/c and whose sine ($\beta_Y = \sin(\theta_1)$) governs the internal ordering (proper time fraction).*
- *S^2 is the minimal geometric structure that can encode omnidirectional relation without absolute space. Its parameter is an angle θ_2 whose sine ($\kappa = \sin(\theta_2)$) we identify with the escape velocity fraction v_e/c and whose cosine ($\kappa_X = \cos(\theta_2)$) governs the internal ordering (proper time fraction).*

This identifies Relativistic and Gravitational Time Dilation not as a mysterious "slowing down" of clocks, but as geometric ***phase rotation****. As a system invests more of its relational existence into external Amplitude (β or κ), it necessarily withdraws from its internal Phase (β_Y or κ_X), changing the rate of its proper time evolution.*

3.2 Kinetic Energy Projection β on S^1

- **The Amplitude Component ($\beta = v/c$):** External shift or the extent of kinematic relation observed from an arbitrary relational origin (the observer's rest frame). It quantifies how much of the universal "transformation rate" is perceived as motion through space, relative to the observer. In legacy units, this is identified as the dimensionless ratio of velocity to the speed of light.
- **The Phase Component ($\beta_Y = \sqrt{1-\beta^2}$):** Internal ordering or the intrinsic scale of proper time and proper length within the moving system. A phase of 1 signifies maximal internal flow (rest), while a phase of 0 indicates the collapse of internal causality (light-speed).

These two components are bound by the conservation theorem (3.6) in closed system, which acts as a finite "budget of transformation":

$$\beta^2 + \beta_Y^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects.

Since S^1 encodes one-dimensional shift, the total energy E of the system must project consistently onto both axes:

$$E^2 = E_X^2 + E_Y^2$$

where

$$E_X = E\beta, \qquad E_Y = E\beta_Y.$$

Desmos project

Kinetic projection β on the S^1 carrier (Special Relativity)

3.2.1 Geometric Nature of Rest Energy State

Theorem 3.9 (Invariant Projection of Rest Energy)**.** *For any state* (β, β_Y) *on the relational circle* S^1*, the total energy* E *must scale such that its vertical projection remains constant and equal to the rest energy* E_0*.*

$$E\beta_Y = E_0.$$

Proof. Let the total energy E be distributed on the relational carrier S^1 with projections β (kinematic amplitude) and β_Y (internal phase).

$$E_Y = E \cdot \beta_Y$$

By Principle 2.2 (Relational Origin), the internal structure of an object (its rest energy E_0) is defined solely by relations internal to itself. Therefore, it must be invariant under changes in its relation to an external observer. In relation to itself kinematic projection is always ($\beta = 0 \implies \beta_Y = 1$). Therefore:

$$\boxed{E_Y \equiv E_0}$$

where E_0 is the energy measured in the frame where the system is at rest ($\beta = 0, \beta_Y = 1$). Thus, geometric consistency requires the vertical leg to be fixed:

$$E\beta_Y = E_0 \quad \implies \quad E = \frac{E_0}{\beta_Y}.$$

The "hypotenuse" (Total Energy E) is therefore not a fixed-length vector that rotates (which would reduce E_Y), but a scalable relational magnitude that grows to preserve the invariant vertical leg E_0 against the closure constraint $\beta_Y = \sqrt{1 - \beta^2}$. □

Summary:

The historical Lorentz factor γ is the reciprocal of β_Y. $\gamma = 1/\beta_Y$

3.2.2 The Geometric and Historical Nature of Mass

Corollary 3.10 (Rest Energy and Mass Equivalence)**.** *This geometry does not require kilograms as units or mass as a concept. Within the normalization* $c = 1$*, the invariant rest energy* E_0 *can be expressed as human-convention-driven parameter of mass in kilograms:*

$$E_0 = m.$$

Proof. From the invariant projection $E\beta_Y = E_0$ and closure of S^1, no additional scaling parameter is required. Hence the conventional bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ reduce to tautologies. Mass is therefore not independent, but the rest-energy invariant itself. □

Summary:

Mass is the invariant projection of total rest energy expressed in anthropocentric units of kilograms.

3.2.3 Energy–Momentum Relation

Proposition 3.11 (Horizontal Projection as Momentum)**.** *On the relational circle, the unique linear geometric projection measure of external amplitude from rest is the horizontal projection* $E_X = E\beta$*; hence*

$$p \equiv E_X \equiv E\beta \quad (c = 1).$$

Proof. The rest state is $(\beta, \beta_Y) = (0, 1)$. A shift measure must (i) vanish at rest, (ii) grow monotonically with $|\beta|$, and (iii) flip sign under $\beta \mapsto -\beta$. The only relational candidate satisfying (i)-(iii) is the horizontal projection $E_X = E\beta$. Thus the identification is necessary and unique within methodological constraints. □

Desmos project

Energy-momentum Triangle

Corollary 3.12 (Energy–Momentum Relation). *With p identified by Proposition 3.11 and $m = E_0$, the closure identity yields*

$$E^2 = p^2 + m^2 \quad (c = 1).$$

Equivalently, upon restoring c,

$$E^2 = (pc)^2 + (mc^2)^2.$$

Proof. By closure, $(E\beta)^2 + (E\beta_Y)^2 = E^2$. Substituting $p = E\beta$ and $m = E_0$ proves the claim. Restoring c is dimensional bookkeeping: $p \mapsto pc$ and $m \mapsto mc^2$, while E remains E, yielding the standard form. □

Remark 3.13 (Geometric Forms). *The same identity may be expressed explicitly in terms of circle coordinates:*

$$E^2 = (\tfrac{\beta}{\beta_Y} E_0)^2 + E_0^2 = \big(\cot(\theta_1)\, E_0\big)^2 + E_0^2.$$

These are equivalent renderings of the same geometric necessity.

Remark 3.14 (Units sanity check - bookkeeping). *Using $\beta = v/c$, the identification $p \equiv E\beta$ gives*

$$pc = E\,\frac{v}{c} \quad \Longrightarrow \quad p = \frac{E\,v}{c^2}.$$

With $E = \frac{1}{\beta_Y} mc^2 = \gamma mc^2$ this reduces to $p = \frac{\beta}{\beta_Y} mc = \gamma mv$, the standard relativistic momentum. No new parameters are introduced.

$\beta = v/c \quad \theta_1 = \arccos(\beta)$	
Algebraic Form	**Trigonometric Form**
$\beta = v/c = \sqrt{1 - \beta_Y{}^2}$	$\beta = cos(\theta_1)$
$\beta_Y = \sqrt{1 - (v/c)^2} = \sqrt{1 - \beta^2}$	$\beta_Y = \sin(\theta_1)$

Table 1: Geometric representation of relativistic effects.

Summary

The energy–momentum relation $E^2 = (pc)^2 + (mc^2)^2$ is a geometric identity of S^1 carrier.

3.2.4 Minkowski Interval as Conservation on S^1

The Minkowski spacetime interval, posited by legacy Special Relativity as a physical metric of a 4D background manifold, mathematically collapses into the invariant phase budget of the kinematic relational carrier (S^1).

The standard interval formulation relies on absolute spatial and temporal coordinates:

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \tag{2}$$

Restricting relative translation to a single spatial degree of freedom (dx) eliminates orthogonal components while maintaining exact algebraic equality:

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 \tag{3}$$

Dividing the relation by the coordinate phase component ($c^2 dt^2$) extracts the dimensionless relational ratios:

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \left(\frac{1}{c}\frac{dx}{dt}\right)^2 \tag{4}$$

Substituting the standard kinematic definition of velocity ($v = dx/dt$) and rearranging the terms produces the constraint of the WILL Relational Geometry:

$$\left(\frac{v}{c}\right)^2 + \left(\frac{d\tau}{dt}\right)^2 = \beta^2 + \beta_Y^2 = 1 \tag{5}$$

This derivation mandates the explicit legacy translation mapping:

- **Kinematic Projection (β):** $\beta \equiv \cos(\theta_1) = \frac{v}{c}$
- **Internal Phase Projection (β_Y):** $\beta_Y \equiv \sin(\theta_1) = \sqrt{1 - \beta^2} = \frac{d\tau}{dt} = \frac{1}{\gamma}$

Epistemological Consequence: Elimination of the Manifold

The Lorentz factor (γ) is mathematically identical to the reciprocal of the internal phase projection $\gamma = \frac{1}{\beta_Y} = \frac{1}{\sin\theta_1}$. Kinematic time dilation ($\beta_Y < 1$) is not a physical deformation of a spatial fabric, but a strict geometric rotation on the S^1 unit circle. The constraint $\beta^2 + \beta_Y^2 = 1$ fully replicates the invariant scalar properties of Minkowski space without requiring a background spatial coordinate system.

3.3 Potential Energy Projection κ on S^2

IMPORTANT:

Throughout this work, S^1 and S^2 are not to be interpreted as spacetime geometries but purely as relational carriers encoding energy conservation. Any reading otherwise is a misinterpretation.

Analogous to S^1, the relational geometry of the sphere S^2 provides orthogonal projections for two aspects of omnidirectional transformation. We define them as follows:

- **The Amplitude Component ($\kappa = \frac{v_e}{c} = \sqrt{\frac{R_s}{r}}$):** The *relational potential measure* between the object and the observer. It corresponds to the *extent* of transformation as potential depth. Same way as we did with β on S^1, we will map κ on S^2 as fraction of speed of light. A value of $\kappa = 1$ denotes *saturation*: the entire relational resource of the system has been allocated into the potential channel. This condition mathematically defines the relational horizon where the escape velocity is equal to speed of light ($v_e = c$) or equivalently as radial distance equal to Schwarzschild radius ($r = R_s$ event horizon).
- **The Phase Component (κ_X):** This projection governs the intrinsic scale of proper length and proper time units within the potential field, corresponding to the internal *sequence* of transformation.

These two components are bound by the conservation theorem (3.6) in closed system, which acts as a finite "budget of transformation":

$$\kappa_X^2 + \kappa^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core gravitational phenomena of modern physics.

Desmos project

Potential projection as κ on S^2 carrier

3.3.1 Potential Meridional Section of S^2

By isotropy, the omnidirectional carrier is S^2, but any radially symmetric exchange reduces to a great-circle meridional section. We therefore work on a unit great circle of S^2 with the parameterization $(\kappa_X, \kappa) = (\cos\theta_2, \sin\theta_2)$.

3.3.2 Consequence: Gravitational Effects

The redistribution of the relational budget between the Phase and Amplitude components directly produces the effects of General Relativity. An increase in the relational amplitude measure (κ, gravitational potential) strictly necessitates a geometric decrease in the measure of the internal structure (κ_X). This geometric trade-off is observed physically as gravitational length contraction and time dilation. Thus, the curved geometry of spacetime is merely the shadow cast by the geometry of conserved relations.

3.3.3 Schwarzschild Metric Interval as Conservation on S^2

This derivation demonstrates how the Schwarzschild metric interval, a cornerstone of General Relativity (GR), algebraically maps directly to the closure identity of the potential carrier, $\kappa^2 + \kappa_X^2 = 1$, within WILL Relational Geometry (RG). This is strictly analogous to how the Minkowski interval maps to the kinematic closure identity $\beta^2 + \beta_Y^2 = 1$.

The Schwarzschild metric describes the spacetime geometry around a spherically symmetric, non-rotating mass M. For a static observer (i.e., not moving radially or angularly, so $dr = 0$, $d\theta = 0$, $d\phi = 0$), the metric interval is:

$$ds^2 = c^2 \left(1 - \frac{R_s}{r}\right) dt^2$$

where $R_s = \frac{2GM}{c^2}$ is the Schwarzschild radius.

The proper time $d\tau$ experienced by the static observer is related to the metric interval by $ds^2 = c^2 d\tau^2$. Substituting this into the Schwarzschild interval yields:

$$c^2 d\tau^2 = c^2 \left(1 - \frac{R_s}{r}\right) dt^2$$

Dividing both sides by the coordinate phase component $c^2 dt^2$, we extract the dimensionless relational ratio (the squared gravitational time dilation factor):

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \frac{R_s}{r}$$

We already know that potential phase component $\kappa_X \equiv \frac{d\tau}{dt}$ governs the proper time in a gravitational field. And potential amplitude can be expressed as $\kappa^2 \equiv \frac{R_s}{r}$.

> Note on Epistemic Hygiene: WILL RG parameters do not depend on GR or any other prior model. In addition WILL R.O.M. sections (sec:relational parameterization, sec:operational , sec:G) we show that the classical gravitational constant (G) embedded in R_s is not a necessary foundational primitive of reality. We will explicitly demonstrate how to fully describe gravitational systems without G and without M.

Substituting these pure relational identities back into the ratio, we arrive at the fundamental conservation identity (3.6) for the S^2 potential carrier:

$$\frac{R_s}{r} + \left(\frac{d\tau}{dt}\right)^2 = \kappa^2 + \kappa_X^2 = 1$$

This derivation mandates the explicit legacy translation mapping:

- **Potential Amplitude Projection (κ):** $\kappa \equiv \sin(\theta_2) = \sqrt{\frac{R_s}{r}}$
- **Internal Phase Projection (κ_X):** $\kappa_X \equiv \cos(\theta_2) = \sqrt{1-\kappa^2} = \frac{d\tau}{dt}$

3.3.4 Epistemological Consequence: Unveiling the Geometric Structure

This exercise demonstrates that the algebraic structure of the Schwarzschild metric's g_{tt} component directly maps to the closure identity of the S^2 potential carrier in WILL RG. The terms in GR's description of spacetime curvature (specifically, the gravitational time dilation factor) are shown to be directly equivalent to the squared potential phase and amplitude projections of WILL RG.

Key Insight

While the Schwarzschild metric *describes* gravitational effects within a pre-existing spacetime manifold, WILL RG *generates* the underlying relational geometry ($\kappa^2 + \kappa_X^2 = 1$) from its foundational ontological principle (**SPACETIME $\equiv$ ENERGY**). This algebraic correspondence indicates that the physical reality captured by GR is a projected phenomenon entirely consistent with the more fundamental relational structure of WILL RG. The geometric identity $\kappa^2 + \kappa_X^2 = 1$ is thus not merely a re-parameterization, but a direct consequence of the removal of hidden assumptions about the separation of structure and dynamics.

3.3.5 Gravitational Tangent Formulation

Just as the relativistic energy–momentum relation can be expressed in terms of the kinematic projection $\beta = v/c$, we may construct its gravitational analogue using the potential projection $\kappa = v_e/c$, where v_e is the Newtonian escape velocity at radius r.

In the kinematic case, with $\beta = \cos\theta_1$, the energy relation can be written as

$$E^2 = \left(\cot\theta_1\, E_0\right)^2 + E_0^2, \tag{6}$$

so that the relativistic momentum is expressed as

$$p = (E_0/c)\cot\theta_1. \tag{7}$$

In full geometric symmetry, the gravitational case follows from $\kappa = \sin\theta_2$. We define the scalable gravitational total energy as

$$E_g = \frac{E_0}{\kappa_X}, \qquad \kappa_X = \sqrt{1-\kappa^2}, \tag{8}$$

and introduce the gravitational analogue of momentum (potential momentum):

$$p_g \;=\; (E_0/c)\tan\theta_2. \tag{9}$$

This yields the exact gravitational energy–momentum relation:

$$E_g^2 = (p_g c)^2 + (mc^2)^2. \tag{10}$$

Summary:

$$\beta = \cos\theta_1, \qquad \kappa = \sin\theta_2,$$

$$\beta \longleftrightarrow \kappa, \qquad \cot\theta_1 \longleftrightarrow \tan\theta_2.$$

Kinematic momentum p and gravitational momentum p_g are thus dual projections of the closed relational geometry, expressed through complementary trigonometric forms.

3.4 Clear Relational Symmetry Between Kinematic and Potential Projections

On the unit kinematic circle (S^1) we parameterize

$$(\beta, \beta_Y) = (\cos\theta_1,\ \sin\theta_1),$$

so that the invariant internal phase projection reads

$$E\,\beta_Y = E_0 \Longrightarrow \boxed{E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sin\theta_1}}, \qquad p = \frac{E}{c}\,\beta = \frac{E_0\,\beta}{\beta_Y} = E_0\cot\theta_1,$$

and therefore $E^2 = (pc)^2 + E_0^2$.

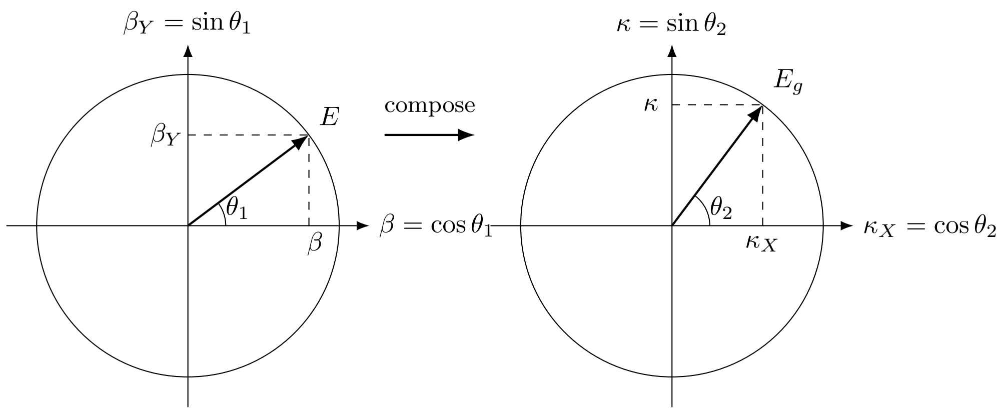
On the gravitational circle (S^2) we parameterize

$$(\kappa_X, \kappa) = (\cos\theta_2,\ \sin\theta_2),$$

so that the invariant internal phase projection reads

$$E_g\,\kappa_X = E_0 \Longrightarrow \boxed{E_g = \frac{E_0}{\kappa_X} = \frac{E_0}{\cos\theta_2}}, \qquad p_g = E_g\,\kappa = \frac{E_0\,\kappa}{\kappa_X} = E_0\tan\theta_2,$$

and therefore $E_g^2 = p_g^2 + E_0^2$.



$E = E_0/\beta_Y$
$p = E_0\,\beta/\beta_Y = E_0\cot(\theta_1)$

$E_g = E_0/\kappa_X$
$p_g = E_0\,\kappa/\kappa_X = E_0\tan(\theta_2)$

Now we can clearly see the underlying symmetry between relativistic and gravitational factors that can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

3.5 Closure: Relational Origin of the Virial Theorem

Having established that directional (kinematic) and omnidirectional (potential) relations are carried by the S^1 and S^2 respectively, we now derive the relationship that unifies them.

3.5.1 Derivation of the Energetic Closure Condition

Remark 3.15 (From slice to whole S^2)**.** *Although we parametrise a single meridional great circle* (κ_X, κ) *for algebraic convenience, the amplitude* κ^2 *denotes the* total omnidirectional budget *of the* S^2 *carrier. The exchange-rate factor* 2 *reflects that* S^2 *has two independent relational degrees of freedom even when calculations are carried out on a representative great-circle section.*

3.5.2 Uniqueness of the Exchange Rate (No Hidden Weighting)

Lemma 3.16 (DOF-Indifference)**.** *Under maximal symmetry (no privileged directions) and ontological minimalism (no hidden structure), any admissible conserved budget must assign* equal quadratic weight *to each independent relational degree of freedom.*

Proof. If two independent DOF contributed unequal weights to the conserved quadratic budget, then the theory would contain an implicit weighting structure that distinguishes DOF. This constitutes a privileged feature not derivable from relations, violating maximal symmetry and minimalism. □

Theorem 3.17 (Closure)**.** *Within WILL, the only exchange rate between the kinematic carrier* S^1 *(1 DOF) and the gravitational carrier* S^2 *(2 DOF) compatible with (i) closure in quadratic form, (ii) maximal symmetry, and (iii) ontological minimalism is:*

$$\boxed{\kappa^2 = 2\beta^2.}$$

Proof. Let b denote the conserved quadratic budget associated with a single independent DOF. By Lemma 3.16, each DOF must contribute the same amount b.

The carrier S^1 has 1 DOF, so its total quadratic budget is $B_{S^1} = 1 \cdot b = b$. The carrier S^2 has 2 DOF, so its total quadratic budget is $B_{S^2} = 2 \cdot b$.

An exchange rate is precisely the statement that the omnidirectional budget is a fixed multiple of the directional budget:

$$B_{S^2} = \mathcal{R}\, B_{S^1}.$$

Substituting $B_{S^2} = 2b$ and $B_{S^1} = b$ gives $\mathcal{R} = 2$, hence

$$\kappa^2 = \mathcal{R}\, \beta^2 = 2\beta^2.$$

Any $\mathcal{R} \neq 2$ would require unequal DOF weighting (hidden structure) or an extra free parameter, violating the methodology. □

Remark 3.18 (Status)**.** *The factor* 2 *is not empirical calibration and not a coordinate choice. It is the unique consequence of (1 DOF vs 2 DOF) under symmetry and minimalism.*

Definition 3.19 (Closure Factor)**.**

$$\delta \equiv \frac{\kappa^2}{2\beta^2}$$

A subsystem is energetically closed *if* $\langle \delta \rangle_{cycle} = 1$. *For circular orbits,* $\delta \equiv 1$.

Corollary 3.20 (Energetic Closure Criterion)**.** *Closed systems (momentary or periodic) satisfy* $\kappa^2 = 2\beta^2$ *identically. Open systems display* $\delta \neq 1$*, the magnitude of which quantifies the energy flow through unaccounted channels. When all channels are included, closure is restored.*

Remark 3.21 (Physical Interpretation)**.** *The exchange rate between the kinematic and gravitational projections corresponds to the ratio of their relational dimensions. This purely geometric constant (2) replaces the empirical proportionalities of classical dynamics. It is the relational origin of the* ***virial theorem****: the kinetic and potential aspects of WILL maintain closure through the invariant ratio*

$$\boxed{\kappa^2 = 2\beta^2.}$$

Illustrative Examples.

- **Circular Orbit (Closed).** A body at any orbital phase satisfies $\kappa^2 = 2\beta^2$. The entire conserved resource is partitioned between kinetic and gravitational projections; no internal "breathing" and no external channel exists.

- **Elliptical Orbit (Closed).** A body satisfies $<\kappa^2>=2<\beta^2>$ as an average per orbital cycle due to internal "breathing" of elliptical systems. Though this internal "breathing" is restricted by the Energy-Symmetry Law (3.6) so the difference $W=\frac{1}{2}(\kappa^2-\beta^2)=$ constant at any orbital phase (**??**). No external channel exists.
- **Radiating Binary (Open).** An elliptical compact binary violates $<\kappa^2>=2<\beta^2>$ when only orbital degrees of freedom are counted, the closure defect δ quantifying energy lost to gravitational radiation. Including all channels restores closure.

Summary:

1. WILL is a closed relational structure, SPACETIME $\equiv$ ENERGY.
2. The simplest maximally symmetric carriers of these relations are S^1 and S^2.
3. The parameters $\beta=\cos\theta_1$ and $\kappa=\sin\theta_2$ are thus constrained to these carriers.
4. The geometric exchange rate between these modes equals the ratio of their DOF: 2.

Remark 3.22 (Geometric Origin of Physical Law). *The relation between kinetic and potential energy is not an empirical coincidence but a geometric necessity of relational closure. Classical mechanics merely approximates this deeper invariant. Explicitly,*

$$\boxed{\textit{Geometric Distribution } (\kappa^2) \;\equiv\; 2\times \textit{Kinetic Distribution } (\beta^2).}$$

Logical Chain Summary

Methodological Constraints
$\downarrow$ *(apply to existing physics)*
False Separation: $\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} + \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}$
$\downarrow$ *(by Relational Origin Principle no backround allowed)*
SPACETIME $\equiv$ ENERGY
$\downarrow$ *(derive consequences)*
Closure + Conservation + Isotropy
$\downarrow$ *(derive primal relational carriers)*
S^1 *(1-DOF kinematic)* + S^2 *(2-DOF potential)*
$\downarrow$ *(By Duality of Relation Lemma)*
Conservation of Relation $\underbrace{\text{Amplitude}^2}_{\text{External Interaction}} + \underbrace{\text{Phase}^2}_{\text{Internal Existence}} = 1$
$\downarrow$ *(only available distinguishing property)*
DOF ratio = 2:1 exchange rate
$\downarrow$ *(By DOF-Indifference Lemma: equal quadratic weight to each independent DOF)*
$\kappa^2=2\beta^2\equiv \underbrace{1\text{Amplitude}^2}_{\text{on } S^2} = \underbrace{2\text{Amplitude}^2}_{\text{on } S^1}$

3.6 Energy-Symmetry Law

In RG, every transformation is bidirectional: each change observed by A corresponds to an equal and opposite change observed by B. This reciprocity is the algebraic form of causal continuity, and its geometric expression is the Energy-Symmetry Law.

3.6.1 Causal Continuity and Energy Symmetry

Theorem 3.23 (Energy Symmetry). *The specific energy differences (per unit of rest energy) perceived by two observers for a transition between their states balance according to the Energy-Symmetry Law:*

$$\Delta E_{A\to B}+\Delta E_{B\to A}=0. \tag{11}$$

Proof. Consider two observers:

- Observer A at rest on the surface at radius r_A (state defined by $\kappa_A, \beta_A=0$).

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- Observer B orbiting at radius $r_B > r_A$ with orbital velocity v_B (state defined by κ_B, β_B).

Each observer perceives energy transfers as the sum of the change in potential and kinetic energy budgets.

From A's perspective (transition from surface to orbit):

1. An object gains potential energy by moving away from the gravitational center.
2. It gains kinetic energy by accelerating to orbital velocity.

The total specific energy required for this transition is the sum of these two contributions:

$$\boxed{\Delta E_{A\to B} = \underbrace{\frac{1}{2}(\kappa_A^2 - \kappa_B^2)}_{\text{Change in Potential}} + \underbrace{\frac{1}{2}(\beta_B^2 - \beta_A^2)}_{\text{Change in Kinetic}}} \tag{12}$$

Since observer A is at rest, $\beta_A = 0$, and the expression simplifies to:

$$\Delta E_{A\to B} = \frac{1}{2}((\kappa_A^2 - \kappa_B^2) + \beta_B^2) \tag{13}$$

From B's perspective (transition from orbit to surface):

1. The object loses potential energy descending into a stronger gravitational field.
2. It loses kinetic energy by reducing its velocity to rest.

This results in a specific energy difference:

$$\Delta E_{B\to A} = \frac{1}{2}((\kappa_B^2 - \kappa_A^2) + \frac{1}{2}((\beta_A^2 - \beta_B^2)) = \frac{1}{2}((\kappa_B^2 - \kappa_A^2) - \beta_B^2) \tag{14}$$

Summing these transfers gives:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0 \tag{15}$$

Thus, no net energy is created or destroyed in a closed cycle of transitions, confirming the Energy-Symmetry Law as a direct consequence of the closed geometry. □

3.6.2 The Specific Energy Transfer (ΔE):

This is the projectional energy difference between states, equivalent to the Total Relational Shift Q, corresponding to the classical total energy of a transition (per unit rest energy). It is defined as the **sum of the changes** in the potential and kinetic energy budgets:

$$\Delta E_{A\to B} = \Delta U_{A\to B} + \Delta K_{A\to B} = \frac{1}{2}(\kappa_A^2 - \kappa_B^2) + \frac{1}{2}(\beta_B^2 - \beta_A^2) \tag{16}$$

It is this quantity, ΔE, that is conserved and must balance to zero in any closed system.

When the closure condition for stable, periodic orbits ($\kappa^2 - 2\beta^2 = 0$) is applied, the general Energy-Symmetry Law simplifies into remarkably elegant and direct forms. These simplified equations provide the precise energy balance for transitions involving energetically closed systems, such as planets or satellites in stable orbits.

Case 1: Surface-to-Orbit Transfer. For a transfer from a state of rest (A, where $\beta_A = 0$) to a closed orbit (B) where E_{0B} is the objects rest energy, the specific energy balance is given by:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\kappa_A^2 - \beta_B^2) \tag{17}$$

This result is derived by applying the closure condition $\kappa_B^2 = 2\beta_B^2$ to the general energy transfer formula, elegantly linking the initial potential projection to the final kinetic projection.

Case 2: Orbit-to-Orbit Transfer. For a transfer between two different closed orbits (A and B), the simplification is even more profound. The specific energy balance reduces to:

$$\frac{E_{A\to B}}{E_{0B}} = \frac{1}{2}(\beta_A^2 - \beta_B^2) \tag{18}$$

In this case, applying the closure condition to both the initial and final orbits causes the potential projection terms (κ^2) to cancel out completely. The entire energy balance of the transfer is expressed purely as the difference between the squares of the initial and final kinetic projections. This demonstrates a deep symmetry in the energetic structure of stable orbital systems.

3.6.3 Physical Meaning of the Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ originate from the quadratic nature of the energy budgets in RG. The energetic significance of a state is proportional to the **square** of its geometric projection. This is the unavoidable consequence of relational carriers closure condition (amplitude^2 + phase^2 = 1). By using only amplitudes (β^2 and κ^2) we operating with half's relational budgets of S^1 and S^2 carriers.

The individual energy budgets are:

- **Specific Potential Energy Budget:** $U/E_0 \propto -\frac{1}{2}\kappa^2$
- **Specific Kinetic Energy Budget:** $K/E_0 = \frac{1}{2}\beta^2$

The factor $\frac{1}{2}$ arises naturally when representing a conserved quantity (energy) through a quadratic measure (the square of a projection). The Energy-Symmetry Law deals with the sum of the *changes* in these individual budgets.

3.6.4 Universal Speed Limit as a Consequence of Energy Symmetry

Theorem 3.24 (Universal Speed Limit)**.** *The universal speed limit ($v \leq c$) emerges naturally from the requirement of energetic symmetry.*

Proof. Assume an object could exceed the speed of light, implying $\beta > 1$. In this scenario, its specific kinetic energy budget, $\frac{1}{2}\beta^2$, would become arbitrarily large.

The energy transfer required to reach this state, $\Delta E_{A\to B}$, would also become arbitrarily large. Consequently, no finite physical process could provide a balancing reverse transfer, $\Delta E_{B\to A}$, that would sum to zero. The fundamental symmetry would be broken:

$$\Delta E_{A\to B} + \Delta E_{B\to A} \neq 0 \tag{19}$$

Therefore, the condition $\beta \leq 1$ (which implies $v \leq c$) is an intrinsic requirement for maintaining the causal and energetic consistency of the relational universe. □

3.6.5 Single-Axis Energy Transfer and the Nature of Light

Theorem 3.25 (Single-Axis Transformation)**.** *For light, the kinematic projection amplitude reaches its full extent:*

$$\boxed{\beta = 1 \Longrightarrow \beta_Y = 0.}$$

This means that all transformation of the relational resource occurs along a ***single X axis*** $\Longrightarrow \beta = 1$*. The orthogonal Y axis is absent* $\Longrightarrow \beta_Y = 0$*, and the total resource of transformation is entirely expressed on one geometric component.*

Proof. For massive bodies, the Energy–Symmetry Law (Section 3.6) partitions the transformation resource equally between orthogonal axes. The specific binding energy invariant for orbital motion is therefore:

$$W_{\text{mass}} = \frac{1}{2}(\kappa^2 - \beta^2), \tag{20}$$

where the factor $\frac{1}{2}$ arises from this dual-axis distribution (Theorem 3.6). This invariant is conserved for closed systems and reduces to the classical Keplerian energy under the closure condition $\kappa^2 = 2\beta^2$ (Corollary 3.20).

Now consider light. By the Single–Axis Transformation Principle (Theorem 3.25), its kinematic projection saturates the carrier:

$$\beta = 1 \Longrightarrow \beta_Y = 0.$$

Geometrically, this collapses the relational architecture:

- The Y-axis (Phase component) vanishes entirely, as $\beta_Y = 0$ indicates no internal state evolution.
- No rest frame exists for self–centering (Section 3.7), eliminating the dual–framing that justifies the $\frac{1}{2}$ partitioning.
- The entire transformation resource concentrates on the single X-axis (Amplitude component).

Consequently, the energy invariant for photon interactions with a massive body (projection κ) must exclude the partitioning factor:

$$W_\gamma = \kappa^2 - \beta^2 = \kappa^2 - 1. \tag{21}$$

This is verified with no gravity state ($\kappa = 0$):

$$W_{\text{mass}} = \tfrac{1}{2}(0^2 - 0^2) = 0,$$
$$W_\gamma = 0^2 - 1^2 = -1.$$

The value $W_\gamma = -1$ corresponds to the full rest energy cost of transitioning to a photon state - whereas the $\frac{1}{2}$ factor would erroneously yield $-\frac{1}{2}$, violating energy symmetry.

For a concrete test case, consider a photon interacting with a massive body where $\kappa_M = 0.4$:

$$W_{\text{mass}} = \tfrac{1}{2}(0.4^2 - 1^2) = \tfrac{1}{2}(-0.84) = -0.42,$$
$$W_\gamma = 0.4^2 - 1^2 = -0.84 = 2 \times W_{\text{mass}}.$$

Thus, the gravitational effect on light is twice that on massive particles - matching the observed factor in light deflection.

The general energy potential follows directly: for light, the full geometric resource expresses unpartitioned along the single axis:

$$\Phi_\gamma = \kappa^2 c^2,$$

while massive bodies retain the partitioned form:

$$\Phi_{\text{mass}} = \tfrac{1}{2}\kappa^2 c^2.$$

This factor-of-2 is the geometric signature of axis count in relational space. No auxiliary approximations or background structures are introduced; the result emerges from the topological constraint $\beta_Y = 0$ applied to the closed carrier S^1. □

Summary

Energy Symmetry Law $\Delta E_{A\to B} + \Delta E_{B\to A} = 0$ - universal relational energy bookkeeping.
The Speed of Light is the boundary beyond which the energy symmetry law breaks down.
Causality is built-in feature of Relational Geometry.
Light has no rest frame. The disappearance of the Phase component (Y-axis $\beta_Y = 0$) concentrates the entire transformation resource on a single geometric component. This eliminates the $\frac{1}{2}$ partitioning factor, yielding $\boxed{\Phi_\gamma = \kappa^2 c^2}$ and explaining why light experiences twice the geometric effect of massive bodies.
This explains the experimentally verified factor of 2 in gravitational lensing and Shapiro delay, traditionally requiring full General Relativity to derive.

3.7 Total Relational Shift Q

Desmos Project

Q as Total Relational Shift

When an observer targets another system, the assigned Total Relational Shift norm Q is defined by the orthogonal projections:

$$\boxed{Q^2 = \beta^2 + \kappa^2} \tag{22}$$

Remark 3.26 (Closure-specific simplification)**.** *Under closure $\kappa^2 = 2\beta^2$ (circular/periodic systems), the norm reduces to $Q^2 = 3\beta^2$ (proved in section 3.5).*

In general (open or elliptic) configurations, the full definition with orbital phase "O" dependence $Q_o(O)^2 = \beta_o(O)^2 + \kappa_o(O)^2$ must be used. See R.O.M. paper for details and derivations.

3.7.1 Relational Reciprocity and Self-Centering

Theorem 3.27 (Relational Reciprocity)**.** *The operation of self-centering $(\beta, \kappa) = (0, 0)$ is a strict geometric necessity of relational origin principle (2.2), causal balance, and the Total Relational Shift Q between two systems is invariant regardless of the observing frame:*

$$Q^2_{A\to B} = Q^2_{B\to A}.$$

Proof. Consider two interacting systems, A and B. By relational origin principle (2.2) for observer A to measure the relational shift of B, system A must serve as the zero-state reference of its own local frame . This geometrically forces the self-centering operation:

$$(\beta_A, \kappa_A) = (0, 0).$$

From this relational origin, A measures the projections of B as (β_B, κ_B), yielding the norm:

$$Q^2_{A\to B} = \beta_B^2 + \kappa_B^2.$$

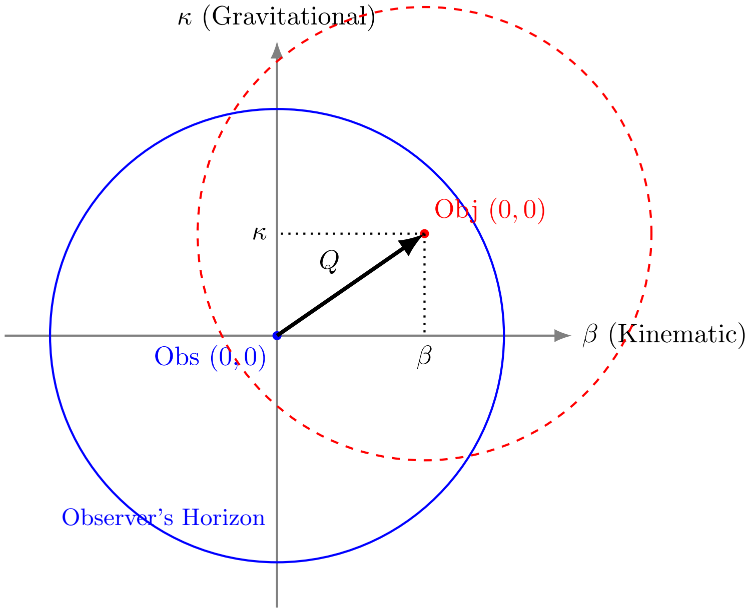


Figure 1: **Relational Self-Centering.** The total shift Q is defined by the orthogonal projections β and κ. Interaction is causal only when the center of the Object lies within the Observer's horizon ($Q < 1$), ensuring mutual coverage.

Conversely, when B observes A, system B constitutes the local zero-state $(\beta_B, \kappa_B) = (0, 0)$. This forces B to measure the exact same total shift norm for A:

$$Q^2_{B\to A} = \beta^2_A + \kappa^2_A = Q^2_{A\to B}.$$

Therefore, reciprocity is not a vector symmetry mapped onto a shared background space. It is the strict invariance of the relational norm Q under the self-centering operation required by relational origin principle (2.2). □

Summary

Relational reciprocity = invariance of the norm Q under self–centering.

There is no common background arena. There are only mutual total shift magnitudes Q computed from each observer's own relational origin.

4 Stage III: Legacy Translation & Empirical Alignment

$\theta_1 = \arccos(\beta)$, $\theta_2 = \arcsin(\kappa)$, $\kappa^2 = 2\beta^2$, a = semi-major axis	
Algebraic Form	**Trigonometric Form**
$\beta = v/c$	$\beta = \cos(\theta_1)$
$\kappa = \sqrt{R_s/a}$	$\kappa = \sin(\theta_2)$
$\beta_Y = \sqrt{1-\beta^2}$	$\beta_Y = \sin(\theta_1)$
$\kappa_X = \sqrt{1-\kappa^2}$	$\kappa_X = \cos(\theta_2)$
$p = E_0/c \cdot \beta/\beta_Y$	$p = E_0/c \cdot \cot(\theta_1)$
$p_g = E_0/c \cdot \kappa/\kappa_X$	$p_g = E_0/c \cdot \tan(\theta_2)$
$\tau = \beta_Y \kappa_X$	$\tau = \sin(\theta_1)\cos(\theta_2)$
$\tau_Y = \sqrt{1-\tau^2}$	$\tau_Y = \sqrt{1-\tau^2}$
$Q = \sqrt{\kappa^2+\beta^2} = \sqrt{3}\beta$	$Q = \sqrt{3}\cos(\theta_1)$
$Q_Y = \sqrt{1-Q^2} = \sqrt{1-\kappa^2-\beta^2} = \sqrt{1-3\beta^2}$	$Q_Y = \sqrt{1-3\cos^2(\theta_1)}$

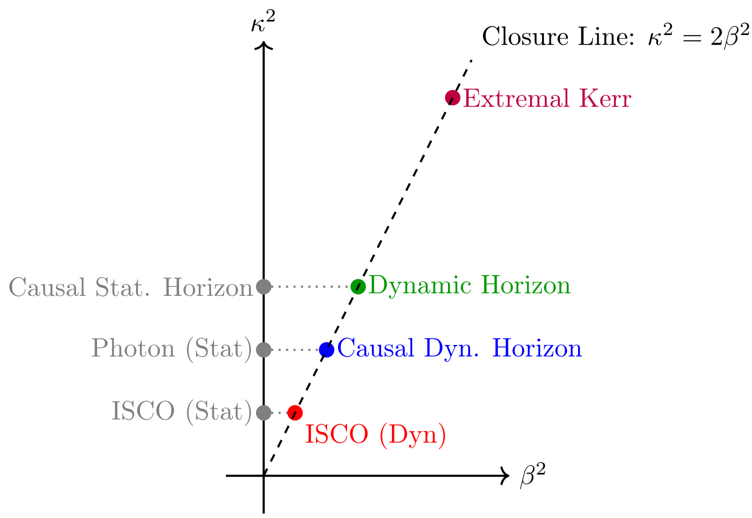
Table 2: Unified representation of relativistic and gravitational effects for closed systems.

Summary

The familiar SR and GR factors emerge here as projections of the same conserved resource. Relativistic (β) and gravitational (κ) modes are not separate "effects" but dual aspects of one energy-transformation constraint revealing their unified origin.

Phenomenon	**Radius** r	β^2	κ^2	Q^2	**Comment**
ISCO (Static location)	$3R_s$	0	$\frac{1}{3}$	$\frac{1}{3}$	Pure coordinate location, $r = R_s/\kappa^2$
ISCO (Dynamic state)	$3R_s$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	Marginal orbital stability, $Q^2 = Q_Y^2 = 1/2$
Photon sphere (Static)	$\frac{3}{2}R_s$	0	$\frac{2}{3}$	$\frac{2}{3}$	Pure coordinate location
Causal Dynamic horizon	$\frac{3}{2}R_s$	$\frac{1}{3}$	$\frac{2}{3}$	1	Relational horizon under closure $\kappa^2 = 2\beta^2$, $Q = 1$
Causal Static horizon	R_s	0	1	1	Purely gravitational closure limit, $\kappa^2 = 1$
Dynamic horizon	R_s	$\frac{1}{2}$	1	$\frac{3}{2}$	Orbital closure at Schwarzschild radius
Extremal Kerr horizon	$\frac{1}{2}R_s$	1	2	3	Maximal rotation, $\beta = 1$, merged horizons

Table 3: Critical radii and their projectional parameters. Static coordinate locations ($\beta = 0$) are strictly distinguished from their corresponding dynamic orbital states that emerge from the closure law $\kappa^2 = 2\beta^2$.



4.1 Equivalence Principle as Derived Identity

Lemma 4.1 (Unified Relational Scaling)**.** *Within the relational framework of WILL, both kinematic (S^1) and gravitational (S^2) transformations act as independent projections of the same invariant energy E_0. Each projection rescales the observable quantities by its respective geometric factor:*

$$E = \frac{E_0}{\beta_Y}, \qquad E_g = \frac{E_0}{\kappa_X}.$$

Proof. On the kinematic circle S^1, the invariant vertical projection corresponds to $\beta_Y = \sin\theta_1$. Preserving the same invariant leg E_0 forces the stretch $E/E_0 = 1/\beta_Y$. On the gravitational sphere S^2, the invariant horizontal projection is $\kappa_X = \cos\theta_2$, forcing $E_g/E_0 = 1/\kappa_X$. These transformations are independent and commute, each preserving the closure identity of its respective carrier. □

Theorem 4.2 (Equivalence of Inertial and Gravitational Response)**.** *Composing the independent relational stretches of Lemma 4.1 yields the total local energy scale*

$$E_{\mathrm{loc}} = \frac{E_0}{\tau} = \frac{E_0}{\beta_Y\,\kappa_X} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}}.$$

The corresponding inertial and gravitational projections share a single operational factor,

$$\tilde{p} = \frac{E_{\rm loc}}{c}\beta, \qquad \tilde{p}_g = \frac{E_{\rm loc}}{c}\kappa,$$

both governed by the same effective mass

$$m_{\rm eff} = \frac{E_0}{\beta_Y\,\kappa_X\,c^2} = \frac{E_0}{\tau c^2}.$$

Therefore,

$$\boxed{m_g \equiv m_i \equiv m_{\rm eff}},$$

and the Einstein equivalence principle follows as a necessary structural identity of WILL.

Corollary 4.3 (Mass Invariance under Relational Scaling)**.** *The invariant core E_0 denotes the complete internal equilibrium state ($\beta_Y = \kappa_X = 1$). Relational factors β_Y and κ_X rescale only external manifestations (energy, momentum, and rates), while E_0 remains unchanged. Hence,*

$$\boxed{m_g \equiv m_i \equiv m = E_0/c^2},$$

is not a dynamical statement but the definition of rest invariance itself.

Remark 4.4 (Composition-Independence)**.** *Decomposing the invariant rest energy into internal channels,*

$$E_0 = \sum_a E_0^{(a)},$$

each term couples identically through the same geometric stretch:

$$E_{\rm loc} = \sum_a \frac{E_0^{(a)}}{\tau}.$$

Since all channels scale by the same factor $1/\tau = 1/(\beta_Y\kappa_X)$, ratios between channels cancel in all observables. Therefore, composition-independence of motion (Eotvos universality) follows identically, without requiring a postulate $m_g = m_i$.

Remark 4.5 (Quantum Interface)**.** *The relational phase increment inherits the same scaling:*

$$\Delta\phi \propto E_{\rm loc}\,\Delta\lambda,$$

where $\Delta\lambda$ is the internal ordering parameter. Thus both kinematic and gravitational phase shifts share the same stretch $1/\tau = 1/(\beta_Y\kappa_X)$, yielding composition-independent matter-wave interference patterns.

Summary:

In WILL, the equivalence of inertial and gravitational mass is not assumed but follows necessarily from the compositional closure of relational geometry. What General Relativity posits as a postulate, WILL reveals as a corollary.

4.2 Classical Mechanics

A striking consequence of the Energy–Symmetry Law (Section 3.6) emerges when analysing the total specific orbital energy. Since energy in RG is defined *relationally, as the measure of difference between two states*, we naturally select these two states (e.g., the surface of the central body 'A' and the orbit 'B') as the reference points for the potential and kinetic energy budgets. Under this relational approach, the total specific orbital energy (potential + kinetic, per unit rest mass) naturally appears in a form **structurally identical to the Minkowski interval**.

4.2.1 Classical Result with Surface Reference

For a test body of mass m on a circular orbit of radius a about a central mass $M_\oplus$ (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \tag{23}$$

$$K = \frac{1}{2}m\frac{GM_\oplus}{a}. \tag{24}$$

Adding these and dividing by the rest–energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\rm tot}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2}\frac{GM_\oplus}{ac^2}. \tag{25}$$

4.2.2 Projection Parameters and Minkowski-like Form

Remark 4.6. *Here we explicitly define our projections using classical language to ease understanding for readers not yet familiar with WILL RG. For more explicit comparison and clearer derivations, we adopt standard Newtonian notation and define the projection parameters κ^2 and β^2 through their legacy equivalents involving G, M, and c. This is purely a pedagogical choice—the projections themselves remain dimensionless geometric quantities independent of these conventional constants.*

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \tag{26}$$

$$\beta_{\text{orbit}}^2 \equiv \frac{GM_\oplus}{ac^2}. \tag{27}$$

Substituting into (4.2.1) gives the exact identity:

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\big(\kappa_\oplus^2 - \beta_{\text{orbit}}^2\big). \tag{28}$$

This is already in the form of a *hyperbolic difference of squares*: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\text{orbit}}$, then

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}\,(x^2 - y^2), \tag{29}$$

which is structurally identical to a Minkowski interval in $(1+1)$ dimensions, up to the constant factor $\frac{1}{2}$.

Sign convention. We use $U/E_0 = -\frac{1}{2}\,\kappa^2$ and $K/E_0 = \frac{1}{2}\,\beta^2$ as *budgets*. The minus sign attaches to the potential budget by convention of reference (surface vs infinity); the budgets themselves are positive quadratic measures, while transfer ΔE is the signed sum of budget *changes*.

4.2.3 Physical Interpretation

In classical derivations, (4.2.1) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the RG, (4.2.2) emerges directly from the energy–symmetry relation:

$$\Delta E_{A\to B} = \frac{1}{2}\big((\kappa_A^2 - \kappa_B^2) + \beta_B^2\big),$$

with $(A, B) = (\text{surface, orbit})$, and is *invariantly* expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure. While RG refuse to postulate any spacetime metric in the traditional sense, the emergence of this Minkowski-like structure from purely energetic principles is a powerful indicator of the deep identity between the geometry of spacetime and the geometry of energy transformation.

Why This Matters

- In classical form, the total orbital energy per unit mass depends only on GM and a, and is independent of the test–mass m.
- In WILL form, the same fact is embedded in Energy–Symmetry difference of squared projections, with no need for separate "gravitational" and "kinetic" constructs.
- This re framing answers *why* the Keplerian combination appears: it is enforced by the underlying geometry of energy transformation.

4.3 Lagrangian Hamiltonian and Newton's Third Law

The following section present philosophical and algebraic demonstration: the standard L and H arise as degenerate limits of the relational Energy-Symmetry law.

We now demonstrate that the familiar Lagrangian and Hamiltonian formalisms arise as limiting cases of the two-point relational Energy-Symmetry Law. Specifically, they emerge when the relational structure between two distinct observers A and B is collapsed into a single-point local description. This collapse preserves computational utility but reduces the ontological transparency of the underlying relational structure.

4.3.1 Definitions of Parameters

We consider a central mass M and a test mass m. The state of the test mass is described in polar coordinates (r, ϕ) relative to the central mass.

- r_A — reference radius associated with observer A (e.g., planetary surface).
- r_B — orbital radius of the test mass m (position of observer B).
- $v_B^2 = \dot{r}_B^2 + r_B^2\dot{\phi}^2$ — total squared orbital speed at B.
- $\beta_B^2 = v_B^2/c^2$ — dimensionless kinematic projection at B.
- $\kappa_A^2 = 2GM/(r_A c^2)$ — dimensionless potential projection defined at A.

4.3.2 The Relational Lagrangian

Instead of a relational energy, we define the *clean relational Lagrangian* L_{rel}, which represents the kinetic budget at point B relative to the potential budget at point A:

$$L_{\text{rel}} = T(B) + U(A) = \tfrac{1}{2}m(\dot{r}_B^2 + r_B^2\dot{\phi}^2) + \frac{GMm}{r_A}. \tag{30}$$

In dimensionless form, using the rest energy $E_0 = mc^2$, this is:

$$\frac{L_{\text{rel}}}{E_0} = \tfrac{1}{2}\big(\beta_B^2 + \kappa_A^2\big). \tag{31}$$

This two-point, relational form is the clean geometric statement.

4.3.3 First Ontological Collapse: The Newtonian Lagrangian

If one commits the first ontological violation by identifying the two distinct points, $r_A = r_B = r$, the relational structure degenerates into a local, single-point function:

$$L(r, \dot{r}, \dot{\phi}) = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \tag{32}$$

This is precisely the standard Newtonian Lagrangian. Its origin is not fundamental but arises from the collapse of the two-point relational Energy Symmetry law into a one-point formalism.

4.3.4 Second Ontological Collapse: The Hamiltonian

Introducing canonical momenta,

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \tag{33}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}, \tag{34}$$

one defines the Hamiltonian via the Legendre transformation $H = p_r\dot{r} + p_\phi\dot{\phi} - L$. This evaluates to the total energy of the collapsed system:

$$H = T + U = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \tag{35}$$

4.3.5 Interpretation

In terms of the collapsed WILL projections ($\beta^2 = v^2/c^2$ and $\kappa^2 = 2GM/(rc^2)$, both strictly positive), the match to standard mechanics becomes explicit:

$$L = \tfrac{1}{2}mv^2 + \frac{GMm}{r} \quad \longleftrightarrow \quad \tfrac{1}{2}mc^2\big(\beta^2 + \kappa^2\big), \tag{36}$$

$$H = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \quad \longleftrightarrow \quad \tfrac{1}{2}mc^2\big(\beta^2 - \kappa^2\big). \tag{37}$$

Here the "+" or "−" signs do not come from κ^2 itself, which is always positive, but from the ontological collapse of the two-point relational energy law into a single-point formalism. In WILL, both projections are clean and positive; in standard mechanics, the apparent sign difference arises only after this collapse.

Both the Lagrangian and Hamiltonian thus emerge from the same relational Energy-Symmetry Law under the identification $r_A = r_B = r$. The apparent sign difference between L and H is not a fundamental feature but an artifact of this single-point collapse: in the full two-point relational law, both projections β^2 and κ^2 are strictly positive.

Remark 4.7 (Mathematical Status: Groupoid vs. Group)**.** *From a category-theoretic perspective, the relational transitions $\Delta E_{A\to B}$ form a **Groupoid** structure, where operations are defined only between specific connected states. Standard field theories (Hamiltonian/Lagrangian) rely on the collapse of this structure into a **Group** acting on a global manifold (Quotienting). Thus, the relational Energy-Symmetry Law operates at the Groupoid level, prior to the reduction into global field theories.*

Key Message

The Lagrangian and Hamiltonian arise as single-point limiting cases of the two-point relational Energy-Symmetry Law $\Delta E_{A\to B} + \Delta E_{B\to A} = 0$. They remain computationally valid within their domain of applicability. RG provides the more general two-point relational structure from which these formalisms can be systematically derived.

Legacy Dictionary (for conventional formalisms).

Within RG, all physical content is expressed purely in terms of relational projections β and κ on S^1 and S^2. For readers accustomed to standard frameworks, the following translation rules may help:

1. *General Relativity (metric form):*

$$\kappa_X \mathrel{\hat{=}} \sqrt{-g_{tt}} \quad \text{(static spacetimes)}, \qquad \beta \mathrel{\hat{=}} \frac{\|u^\mu_{\text{spatial}}\|}{u^t c}.$$

2. *Canonical mechanics (Lagrangian/Hamiltonian):* Quantities such as $p_i = \partial L/\partial \dot{q}^i$ do not belong to the ontology of RG. They arise only after collapsing the two-point relational law into a one-point formalism. They are secondary derived quantities arising after the two-point relational structure is reduced to a local single-point formalism.

Here the symbol $\hat{=}$ denotes not an ontological identity, but a pragmatic dictionary entry for translation into legacy notation.

4.3.6 Third Ontological Collapse: Newton's Third Law

We now demonstrate that Newton's Third Law, like the Lagrangian and Hamiltonian, emerges as a limiting case of RG. It arises as a necessary mathematical consequence of the same ontological collapse - forcing a two-point relational law into a single-point, instantaneous formalism.

Theorem 4.8 (Newton's Third Law as a Single-Point Limit)**.** *The Energy–Symmetry Law ($\Delta E_{A\to B} + \Delta E_{B\to A} = 0$) mathematically necessitates Newton's Third Law ($\vec{F}_{AB} = -\vec{F}_{BA}$) in the classical, non-relativistic limit where the two-point relational energy budget is collapsed into a single-point potential function $U(\vec{r})$.*

Proof. We begin with the foundational Energy–Symmetry Law (Section 3.6), the principle of causal balance for state transitions:

$$\Delta E_{A\to B} + \Delta E_{B\to A} = 0.$$

In the classical, non-relativistic limit, this two-point relational law is "ontologically collapsed" into a single-point potential energy function, U. This function is assumed to depend only on the relative positions of the two interacting entities, A and B:

$$U = U(\vec{r}) \quad \text{where} \quad \vec{r} = \vec{r}_B - \vec{r}_A.$$

This $U(\vec{r})$ is the classical approximation of the system's relational energy budget. In this collapsed formalism, the force $\vec{F}$ is defined as the negative gradient of this potential.

(1) Force on B by A ($\vec{F}_{AB}$): This force is found by taking the gradient with respect to B's coordinates:

$$\begin{aligned}
\vec{F}_{AB} &= -\nabla_B U(\vec{r}_B - \vec{r}_A) && (38)\\
&= -(\frac{dU}{d\vec{r}}) \cdot (\frac{\partial \vec{r}}{\partial \vec{r}_B}) && (39)\\
&= -\nabla U(\vec{r}) \cdot (\mathbf{I}) && (40)\\
&= -\nabla U(\vec{r}) && (41)
\end{aligned}$$

(2) Force on A by B ($\vec{F}_{BA}$): This force is found by taking the gradient with respect to A's coordinates:

$$\vec{F}_{BA} = -\nabla_A U(\vec{r}_B - \vec{r}_A) \tag{42}$$
$$= -(\frac{dU}{d\vec{r}}) \cdot (\frac{\partial \vec{r}}{\partial \vec{r}_A}) \tag{43}$$
$$= -\nabla U(\vec{r}) \cdot (-\mathbf{I}) \tag{44}$$
$$= +\nabla U(\vec{r}) \tag{45}$$

(3) Conclusion: By direct comparison of the results, we find:

$$\vec{F}_{AB} = -\nabla U(\vec{r}) \quad \text{and} \quad \vec{F}_{BA} = +\nabla U(\vec{r}).$$

Therefore, within the single-point collapsed formalism:

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$

This completes the proof. Since all physical observations are strictly relative, embedding the energy budget within an absolute spatial container ($\vec{r}_A, \vec{r}_B$) introduces unobservable parameters, which strictly violates the principle of Epistemic Hygiene. To remain empirically grounded, the collapsed potential U must depend exclusively on the measurable physical relation $\vec{r}_B - \vec{r}_A$. Consequently, the principle of equal and opposite forces emerges not as an empirical axiom of mechanics, but as the mathematical signature of any causally balanced, background-independent system $\Delta E_{A\to B} + \Delta E_{B\to A} = 0$. □

4.3.7 Fourth Ontological Collapse: The Einstein-Hilbert Action (S_{EH})

The Einstein-Hilbert action ($S_{EH} = \frac{1}{16\pi G}\int R\sqrt{-g}\, d^4x$) represents a reduction of relational connections between states into localized coordinate points. Standard tensor calculus employs the limit $dx \to 0$ to compress relational tension into the local Ricci scalar (R). This operation necessitates the postulation of a smooth background manifold and the application of the Principle of Stationary Action ($\delta S_{EH} = 0$) to maintain global equilibrium across arbitrary coordinates.

4.3.8 Components Mapping

- **Global 4D Volume** ($\int \sqrt{-g}\, d^4x \to 1$): Instead of summation over a background manifold, relational capacity in RG is bound by the carriers $\kappa^2 + \kappa_X^2 = 1$ and $\beta^2 + \beta_Y^2 = 1$(3.6).
- **The Ricci Scalar** ($R \to \kappa^2, \beta^2$): Differential approximation replaced with relational projections.
- **The Variational Search** ($\delta S = 0 \to \Delta E_{A\to B} + \Delta E_{B\to A} = 0$): Reduced to the Law of Energy Symmetry ($\Delta E_{A\to B} + \Delta E_{B\to A} = 0$) (3.6).

Empirical Grounding

of the action (S_{EH}) manifests in gravitational light deflection via the addition of a curved spatial metric (g_{rr}). RG achieves the identical empirical result without introducing new ontological primitives: (Gravitational Deflection and Lensing).

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4.4 Earth–GPS System: Standard GR as First-Order Approximation of RG

4.4.1 Statement of the Verification Problem

The standard post-Newtonian expression for the secular time shift between a clock on the Earth's surface and a clock on a circular GPS orbit, accumulated over one solar day, is

$$\Delta t_{\mathrm{GR}} = \left(\frac{\Phi_{\mathrm{GPS}} - \Phi_E}{c^2} - \frac{v_{\mathrm{GPS}}^2 - v_E^2}{2\,c^2} \right) D\, M, \tag{46}$$

with $D = 86{,}400\,\mathrm{s}$, $M = 10^6$, $\Phi(r) = -GM_\oplus/r$, $v_E = 2\pi R_E/D$, and $v_{\mathrm{GPS}} = \sqrt{GM_\oplus/r_{\mathrm{GPS}}}$.

WILL Relational Geometry (WILL RG) expresses the same time shift as an exact algebraic ratio of two-carrier projection invariants $\tau \equiv \sqrt{1-\kappa^2}\,\sqrt{1-\beta^2}$:

$$\Delta t_{\rm RG} \;=\; \left(1 \;-\; \frac{\sqrt{1-\kappa_E^2}\,\sqrt{1-\beta_E^2}}{\sqrt{1-\kappa_{\rm GPS}^2}\,\sqrt{1-\beta_{\rm GPS}^2}}\right) D\,M, \tag{47}$$

where $\kappa^2(r) = 2GM_\oplus/(rc^2)$, $\beta^2 = v^2/c^2$, and the orbital closure condition $\beta_{\rm GPS}^2 = \kappa_{\rm GPS}^2/2$ holds for circular motion.

The question addressed in this section is whether (46) is the leading-order Taylor expansion of (47) in the small parameters κ^2 and β^2. If so, the algebraic difference of the two coefficients must vanish identically at first order, and the residual at finite values of the small parameters must reproduce the analytically derived second-order correction.

4.4.2 Methodology

The verification is split into two independent parts, both executed in a single public notebook (Section 4.4.5).

Part I — symbolic identity. Introduce the dimensionless geometric ratio $\rho \equiv R_E/r_{\rm GPS}$, which is treated as a finite (order-unity) parameter and is *not* expanded. The exact WILL RG shift coefficient is then

$$\delta_{\rm RG}(\kappa_E^2, \beta_E^2, \rho) \;=\; 1 - \frac{\sqrt{(1-\kappa_E^2)(1-\beta_E^2)}}{\sqrt{(1-\kappa_E^2\rho)(1-\kappa_E^2\rho/2)}}. \tag{48}$$

A single bookkeeping parameter ε is introduced via $\kappa_E^2 \to \varepsilon\,\kappa_E^2$, $\beta_E^2 \to \varepsilon\,\beta_E^2$, and the Taylor series of (48) is computed to second order in ε using `sympy.series`. The first-order coefficient $\delta_{\rm RG}^{(1)}$ is then subtracted from the dimensionless form of the GR coefficient $\delta_{\rm GR}$ obtained from (46) under the substitutions $\Phi/c^2 = -\kappa^2/2$ and $v^2/c^2 = \beta^2$. The result is simplified algebraically.

Part II — numerical comparison. Both (46) and (47) are evaluated with 40-digit precision arithmetic (`mpmath`, `mp.dps = 40`) using a single set of input parameters: $c = 299{,}792{,}458\,{\rm m\,s^{-1}}$, $GM_\oplus = 3.986004418\times10^{14}\,{\rm m^3\,s^{-2}}$, $R_E = 6{,}378{,}137\,{\rm m}$, $r_{\rm GPS} = 26{,}561{,}750\,{\rm m}$, $D = 86{,}400\,{\rm s}$. The closure residual $\beta_{\rm GPS}^2 - \kappa_{\rm GPS}^2/2$ is monitored as a numerical consistency check. The numerical discrepancy $\Delta t_{\rm RG} - \Delta t_{\rm GR}$ is then compared with the analytically predicted second-order correction $\delta_{\rm RG}^{(2)}\,D\,M$ obtained from Part I.

4.4.3 Results

Symbolic. The first-order Taylor coefficient of the WILL RG shift, expressed in κ_E^2, β_E^2, and the geometric ratio ρ, is

$$\delta_{\rm RG}^{(1)} \;=\; \frac{\kappa_E^2}{2} + \frac{\beta_E^2}{2} - \frac{3\,\kappa_E^2\,\rho}{4}. \tag{49}$$

The GR coefficient, expressed in the same variables, is

$$\delta_{\rm GR} \;=\; \frac{\kappa_E^2}{2} + \frac{\beta_E^2}{2} - \frac{3\,\kappa_E^2\,\rho}{4}. \tag{50}$$

The algebraic difference $\delta_{\rm RG}^{(1)} - \delta_{\rm GR}$ vanishes identically.

The second-order content of (48), which is absent from the additive GR formula (46), is

$$\delta_{\rm RG}^{(2)} \;=\; \frac{\kappa_E^4}{8} + \frac{\beta_E^4}{8} - \frac{\beta_E^2\,\kappa_E^2}{4} + \frac{3\,\kappa_E^4\,\rho}{8} - \frac{19\,\kappa_E^4\,\rho^2}{32} + \frac{3\,\beta_E^2\,\kappa_E^2\,\rho}{8}. \tag{51}$$

Three structural features of (51) have no analogue in any rearrangement of the additive GR sum (46): (i) the pure quadratic contributions $\kappa_E^4/8$ and $\beta_E^4/8$ arising from higher-order expansion of $\sqrt{1-x}$; (ii) the cross-term $-\beta_E^2\kappa_E^2/4$, which originates from the multiplicative coupling of the gravitational and kinematic projection factors within $\tau_E/\tau_{\rm GPS}$; and (iii) the ρ^2 contribution, which encodes nonlinear dependence on the relative depth of the gravitational well.

Numerical. With the parameter set listed above:

quantity	value $[\mu{\rm s\,day^{-1}}]$
$\Delta t_{\rm RG}$ (exact)	38.5421472752
$\Delta t_{\rm GR}$ (first order)	38.5421472451
$\Delta t_{\rm RG} - \Delta t_{\rm GR}$	3.017×10^{-8}
$\delta_{\rm RG}^{(2)}\cdot D\,M$ (predicted)	3.017×10^{-8}
residual	$\sim 10^{-17}$

The numerical discrepancy $\Delta t_{\text{RG}} - \Delta t_{\text{GR}}$ matches the analytically derived second-order correction $\delta^{(2)}_{\text{RG}}\, D\, M$ to within the limit set by the 40-digit floating-point arithmetic. The closure residual $\beta^2_{\text{GPS}} - \kappa^2_{\text{GPS}}/2$ vanishes to $\sim 10^{-50}$. The relative magnitude of the term omitted by the GR additive formula is $\sim 7.8 \times 10^{-10}$ of the total shift.

4.4.4 Interpretation

Equations (49)–(50) establish that the additive post-Newtonian formula (46) coincides exactly with the linear Taylor coefficient of the WILL RG exact algebraic ratio (47). This is an algebraic identity in $(\kappa^2_E, \beta^2_E, \rho)$, not a numerical near-match: the additive sum-of-corrections structure of (46) is recovered as the unique leading-order content of the multiplicative ratio τ_E/τ_{GPS}.

For the Earth–GPS configuration, the second-order term (51) contributes at the level of $\sim 3 \times 10^{-8}\,\mu\text{s}\,\text{day}^{-1}$, i.e. $\sim 8 \times 10^{-10}$ of the total accumulated shift. This is below the present precision of operational GPS clock comparisons and does not, by itself, constitute a distinguishing observation between the two formulations in this regime. The structural difference between an additive and a multiplicative composition of projection factors — in particular the presence of the $\beta^2\kappa^2$ cross-term in (51) — becomes quantitatively relevant only when κ^2 or β^2 approach order unity.

The scope of the present statement is restricted to the relation between the WILL RG closed-form expression (47) and the *additive post-Newtonian* form of the GR time-dilation formula (46). A comparison against fully non-linear geodesic integration in the exact Schwarzschild geometry combined with special-relativistic kinematics for the rotating Earth surface is a separate computation and is not addressed in this section.

4.4.5 Reproducibility

The complete symbolic and numerical verification reported in this section is contained in a single public Google Colab notebook, comprising the two scripts used to produce (49), (51), and the numerical table above:

Colab Notebook

Earth-GPS 1st order RG=GR.ipynb

The notebook has no external dependencies beyond `sympy` and `mpmath` and is self-contained. All input parameters are listed explicitly in the numerical script; any change in r_{GPS}, R_E, or $GM_\oplus$ rescales both Δt_{RG} and Δt_{GR} consistently and does not affect the symbolic identity $\delta^{(1)}_{\text{RG}} \equiv \delta_{\text{GR}}$.

4.5 Relational Orbital Mechanics (R.O.M.)

Relational Orbital Mechanics (R.O.M.)

Natural evolution of this section within WILL RG Open Research lead it to branch in to its own paper Full R.O.M. paper

You can access the paper using the link above, and for quick introduction here's the content of the R.O.M. paper:

Contents

Relational Orbital Mechanics as algebraically closed system of equations developed in this section demonstrates that the entire phenomenology of motion – from Keplerian architecture to perihelion precession and light deflection – emerges as an inevitable algebraic consequence of the Foundational Methodological Principles. All WILL RG results including the complete success of R.O.M. remains an unbroken chain of derivations from firs principles under extreme methodological constrains with zero flexibility.

4.6 Density, Mass, and Pressure

4.6.1 Derivation of Density

Translating RG (2D) to Conventional Density (3D). In RG κ^2 is the 2D parameter defined in the relational carrier S^2 . In conventional physics, the source term is volumetric density ρ, a 3D concept defined by the "cultural artifact" (a Newtonian "cannonball" model) of mass-per-volume .

To compare with measurements that are historically expressed in 3D terms, we need to bridge our 2D theory with 3D units that empirical data is using. We must create a "translation interface". We do this by adopting the conventional (Newtonian) definition of density, $\rho \propto M/r^3$, as our "translation target".

From the projective analysis established in the previous sections:

$$\kappa^2 = \frac{R_s}{r},$$

where κ emerges from the energy projection on the area of unit sphere S^2, and $R_s = 2GM/c^2$ links to the mass scale factor $M = E_0/c^2$.

This leads to mass definition:

$$M = \frac{\kappa^2 c^2 r}{2G}$$

To translate this into a volumetric density, we first adopt the conventional 3D (volumetric) proxy, r^3. This is not a postulate of RG, but the first step in applying the legacy (3D) definition of density:

$$\frac{M}{r^3} = \frac{\kappa^2 c^2}{2Gr^2}$$

This expression, however, is incomplete. Our κ^2 "lives" on the 2D surface S^2 (which corresponds to 4π), while the r^3 proxy assumes a 3D volume (which corresponds to $4\pi/3$). To correctly normalized the 2D parameter κ^2 against the 3D volume, we must apply the geometric normalization factor of the S^2 carrier by dividing on to area of the sphere, which is $1/4\pi$ (we explicitly reject the Newtonian volume integration ($4\pi/3$) in favor of the S^2 carrier's geometric surface (4π). This enforces the relational principle that structural capacity scales exclusively via the 2D spherical boundary of interaction, not by filling an assumed 3D volumetric container).

This normalization is the necessary geometric step to interface the 2D relational carrier (S^2) with the 3D legacy definition of density:

$$\rho = \frac{1}{4\pi}(\frac{\kappa^2 c^2}{2Gr^2})$$

$$\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$$

Local Density $\propto$ Relational Potential Projection

Maximal Density. At $\kappa^2 = 1$ (the horizon condition (for non rotating systems), $r = R_s$), this density reaches its natural bound, $\rho_{\max}$:

$$\rho_{\max} = \frac{c^2}{8\pi G r^2}$$

Normalized Relation. Thus, our "translation" reveals an identity: the potential projection κ^2 is the ratio of density to the maximal density:

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \quad \rightarrow \quad \kappa^2 \equiv \Omega}$$

Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$M = \frac{\kappa^2 c^2 r}{2G}.$$

From the energy density:

$$M = \alpha\, r^n\, \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$ into $M = \alpha\, r^n\, \rho$ gives

$$M = \frac{\alpha \kappa^2 c^2 r^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r^{n-2}}{8\pi} = \frac{r}{2}.$$

For the mass M to remain a constant independent of the measurement scale r, the exponent must be $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$M = 4\pi r^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

Desmos Project

Derivation of Density and Pressure

4.6.2 Pressure as Surface Curvature Gradient

In the RG framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) \;=\; \frac{c^4}{8\pi G}\,\frac{1}{r}\,\frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) \;=\; -\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) \;=\; \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$\boxed{P(r) = -\,\rho(r)\,c^2}.$$

Interpretation. P is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energy-geometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of κ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Desmos Project

Derivation of Density and Pressure

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\,\rho_{\max}\, c^2 \;=\; -\,\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale r.

Pressure in WILL is the intrinsic surface tension of energy-geometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

Physical meaning

The negative pressure is not an exotic substance but the geometric tension required to maintain self-consistency when κ^2 varies radially. It's the relational analogue of surface tension in a soap bubble.

4.7 Unified Geometric Field Equation

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation linking the geometric scale to the energy density ratio:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho_{field}}{\rho_{max}}}$$

This identity defines the **local energy state of the geometry itself**. Here $\rho_{\max} = c^2/(8\pi G r^2)$ is the saturation density limit, and ρ_{field} is the effective energy density of the relational curvature.

4.8 Field Equation and Matter Sources

For a static, spherically symmetric configuration containing matter with density $\rho_{\text{matter}}(r)$, the relationship is governed by the differential accumulation of the potential:

$$\boxed{\frac{d}{dr}(r\,\kappa^2) = \frac{8\pi G}{c^2}\, r^2 \rho_{matter}(r)} \tag{52}$$

This expression reproduces the tt-component of the Einstein field equations.

The Vacuum Solution ($\rho_{matter} = 0$). In the vacuum region outside a central mass, the source density vanishes ($\rho_{matter} = 0$). The field equation implies conservation of the projection budget:

$$\frac{d}{dr}(r\kappa^2) = 0 \quad \Longrightarrow \quad r\kappa^2 = \text{const} = R_s.$$

Thus, we recover the potential law of WILL RG:

$$\boxed{\kappa^2 = \frac{R_s}{r}.}$$

Resolution of Roles

1. **The Identity** $\kappa^2 = \rho/\rho_{\max}$ describes the state of the *field* geometry.
2. **The Equation** $(r\kappa^2)' \sim \rho_{\text{matter}}$ describes how *matter* generates geometry.

In vacuum, the generator is zero, but the field persists as the algebraic structure $\kappa^2 = R_s/r$.

4.9 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r} = \frac{8\pi G}{c^2} r^2 \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

WILL Relational Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but non-singular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

- **Surface-scaled closure (vs. volume filling).** Mass follows the algebraic closure $M = 4\pi r^3 \rho$ with $\rho = \kappa^2 c^2/(8\pi G r^2)$; the 4π is the spherical projection measure, not a Newtonian volume average.
- **Natural bounds.** The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{\max}$ and $|P| \leq |P_{\max}| = c^4/(8\pi G r^2)$, avoiding singularities without extra hypotheses.

5 Stage IV Consequences

5.1 Theoretical Ouroboros

Closure

Ontological principle is proven as the inevitable consequence of geometric consistency. Field Equation $\iff$ Ontological Principle

We have shown that this single Ontological Principle (2.12), through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with SPACETIME ≡ ENERGY, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and spacetime, proving the self-consistency of the initial idea.

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY.}}$$

The ratio of geometric scales equals the ratio of energy densities.

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}}$$

$$\boxed{\begin{array}{c}\text{SPACETIME GEOMETRY}\\ \equiv \\ \text{ENERGY DISTRIBUTION}\end{array}}$$

Summary

All physical structure emerges from the single relational equivalence:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

From this, by enforcing geometric self-consistency, one necessarily arrives at the Unified Geometric Field Equation:

$$\boxed{\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{\max}}.}$$

This is not an external law but an intrinsic closure relation: geometry and energy are two mutually defining projections of a single entity. It represents the completion of the theoretical Ouroboros — where the principle generates its own mathematical expression and the expression in turn validates the principle.

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework - the "Theoretical Ouroboros". But regardless of the aesthetic beauty of this result, let us remain sceptical.

5.1.1 W_{ILL}: Unity of Relational Structure

The ontological principle

$$\text{SPACETIME} \equiv \text{ENERGY}$$

states that there is only one closed relational resource - WILL. What we call space, time and energy are different projections of the same relational structure. For any energy-closed system observed from a relational origin, this resource appears through four operational projections:

> Desmos project
>
> **WILL= 1** Derivation of the W_{ILL} invariant.

These quantities are defined as correlated projections of the same underlying WILL structure. In the dimensionful form we write:

$$M = \frac{\beta^2}{\beta_Y}\frac{c^2 a}{G}, \qquad E = \frac{\kappa^2}{\kappa_X}\frac{c^4 a}{2G}, \qquad T = \kappa_X(\frac{2Gm_0}{\kappa^2 c^3})^2, \qquad L = \beta_Y(\frac{Gm_0}{\beta^2 c^2})^2,$$

where (β, β_Y) and (κ, κ_X) are the kinematic and gravitational projections on the carriers S^1 and S^2, m_0 is the rest mass, $E_0 = m_0 c^2$ is the rest energy, and a is the relational scale of the system as semi-major axis (average length per cycle within this system).

The closure conditions of the carriers,

$$\beta^2 + \beta_Y^2 = 1, \qquad \kappa_X^2 + \kappa^2 = 1,$$

together with the energetic exchange condition

$$\kappa^2 = 2\beta^2,$$

fix a unique dimensionless combination of these four projections. Combining E, T, M, and L we obtain

$$W_{\mathrm{ILL}} \equiv \frac{ET}{ML} = \frac{\frac{E_0}{\kappa_X}\kappa_X t^2}{\frac{m_0}{\beta_Y}\beta_Y a^2} = \frac{E_0 t^2}{m_0 a^2}.$$

By the relations that tie temporal and $t = a/c$ and spacial $a = R_s/\kappa^2$ scales this ratio is identically equal to unity for any closed system:

$$\boxed{W_{\mathrm{ILL}} = \frac{ET}{ML} = 1.}$$

All dimensionful constants cancel automatically; the value is fixed by the geometry of the carriers, not by a choice of units.

> Desmos project
>
> **WILL EARTH GPS** calculates GPS satellite time shift and testing $W_{ILL} = 1$ on the Earth-GPS system

The same invariant can be written in a phase–normalized form, using local projections

$$E_o = \frac{E_0}{\kappa_{Xo}}, \qquad M_o = \frac{m_0}{\beta_{Yo}}, \qquad T_o = \kappa_{Xo} t_o^2, \qquad L_o = \beta_{Yo} r_o^2,$$

Equivalently, for any state (β_o, κ_o), any scale r_o and any phase along the orbit. Then

$$\boxed{W_{\mathrm{ILL}} = \frac{E_o T_o}{M_o L_o} = 1 \quad \text{for ALL ENERGY'S, ALL SCALES and ALL PHASES.}}$$

$$\frac{E_o}{M_o} = \frac{L_o}{T_o},$$

so the energy sector and the spacetime sector are not independent. Every change of the relational state rescales (E_o, M_o) and (T_o, L_o) coherently so that this equality is always preserved. The familiar practice of treating energy–mass and space–time as separate blocks is therefore an ontological approximation: in WILL they are locked by a single relational constraint.

$$\text{Geometry} \equiv \text{Energy} \equiv \text{Causality} \equiv WILL,$$

$$\boxed{W_{ILL} = 1.}$$

in the precise sense that one and the same conserved relational resource appears as mass, energy, time and length, but always in a way that keeps their ratio fixed.

5.1.2 Interpretive Note: The Name "WILL"

The term **WILL** stands for **SPACE-TIME-ENERGY**. It is both a formal shorthand and a philosophical statement: the universe is not a stage where energy acts through time upon space, but a single self-balancing structure whose internal distinctions generate all phenomena. The name also serves as a gentle irony toward anthropic thinking: the Cosmos does not possess "will" - yet through WILL, it manifests All that Is.

Summary

$$\boxed{\textbf{WILL} \;\equiv\; \frac{ET}{ML} = 1 \quad \Longleftrightarrow \quad \text{Geometry} = \text{Energy} = \text{Causality.}}$$

WILL is not the unit of something - but the Unity of Everything.

5.2 Ontological Shift: From Descriptive to Generative Physics

In conventional physics the methodology follows a descriptive paradigm:

1. Observable phenomena are identified.
2. Empirical regularities are codified as “laws of nature.”
3. Mathematical formalisms are constructed to *describe* these regularities.

Thus, physical laws are always introduced as external assumptions that model what is seen. Even in General Relativity, where geometry plays the central role, the equivalence principle and the metric postulate are still external inputs.

The RG framework inverts this paradigm. Laws are not added on top of observations; they are *generated* as inevitable consequences of relational geometry:

- There are no independent axioms such as “inertial mass equals gravitational mass.”
- Such relations appear automatically as algebraic identities enforced by the geometry.
- What classical physics calls “laws of nature” are secondary shadows of the single relational principle:

$$\text{SPACETIME} \;\equiv\; \text{ENERGY}.$$

Summary

Standard Physics: Laws *describe* what we observe.
Relational Geometry: Laws are *generated* as necessary products of closure and self-consistency.

In this sense, the ontological role of physical law is transformed. Physics ceases to be a catalog of empirical descriptions, and becomes the logical unfolding of a single relational structure. WILL identifies the necessary conditions under which all observed phenomena arise.

Descriptive Physics (Standard)	**Generative Physics (WILL)**
Phenomena are *observed* first, then summarized into empirical laws.	Laws emerge as *inevitable consequences* of relational geometry.
Physical laws are *assumptions* introduced to model reality.	Physical laws are *identities*, enforced by geometric self-consistency.
Time and space are treated as external backgrounds.	Time and space are *projections of energy relations.*
Dynamics = evolution of states *in time.*	Dynamics = ordered succession of balanced configurations; *time is emergent.*
Goal: *describe* what is observed.	Goal: *show why nothing else is possible.*

Table 4: Ontological contrast between standard descriptive physics and the generative paradigm of WILL Relational Geometry.

Phenomenon	**Standard (GR) Result**	**Relational Geometry (RG)**
GPS time shift / gravitational redshift	Frequency shift = combination of kinetic (SR) and gravitational (GR) effects.	Single symmetric law: $\tau = \beta_Y \cdot \kappa_X$, $E_{\text{loc}} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}} = \frac{E_{\text{loc}}}{\tau}$ 38.52 μs/day verified directly with GPS satellites.
Photon sphere, ISCO, horizons	Derived by solving geodesic equations in Schwarzschild metric.	Critical radii emerge from simple symmetry's (Photon sphere: $\theta_1 = \theta_2 = 54.73°$ ("magic angle") $Q^2 = \kappa^2 + \beta^2 = 1$. ISCO: $Q = Q_Y$,
Mercury's perihelion precession	Complex expansion of Einstein field equations.	Same number obtained from RG with $\Delta\varphi = \frac{2\pi \cdot \tau_Y^2}{1-e^2} = 43''$/century. Using simle algebra.
Cosmological redshift	Photon "loses energy" as universe expands.	Energy conserved; redshift = redistribution of projection parameters. (Details in WILL PART II)
Cosmological absolute scale (Supernovae fit)	Hubble-like expansion, ΛCDM fits	$H_0 \equiv \sqrt{8\pi G\rho_\gamma/(3\alpha^2)} = 68.15$ Derived from CMB temperature and α connecting micro and macro scales (Details in WILL PART II-III).
Cosmological constant Λ	Added by hand to fit data ("dark energy").	Arises naturally as $\Lambda = 2/3r^2$. No extra entities required. (More details in WILL PART I and II)
Singularities	Predicted in black holes and big bang ($\rho \to \infty$).	Forbidden: density bounded by $\rho_{\max} = c^2/(8\pi G r^2)$.
Local gravitational energy	"Cannot be localized" (only ADM/Bondi at infinity).	Directly measurable via κ, e.g. from light deflection angle or red shift.
Unification with QM	No natural unification in GR framework.	Same projectional law applies from microscopic $\alpha = \beta_1$ (QM) to cosmic $\kappa^2 = \Omega_\Lambda$ (GR, COSMO) scales. (Details in WILL PART II and III)

Table 5: Classical GR results vs. WILL RG outcomes. Known effects are recovered by simpler symmetric laws, while new predictions eliminate singularities and explain cosmology without dark entities.

5.3 Conclusion

This open research demonstrates that the predictive content of Special and General Relativity can be systematically recovered by dissolving the historical separation between "structure" and "dynamics." By adhering to the constraints of Epistemic Hygiene and Ontological Minimalism, we show that the laws of physics are not external models imposed upon reality, but are the necessary, generated consequences of a single relational identity: $SPACETIME \equiv ENERGY$.

The results of Part I establish several firm milestones for Relational Geometry (RG):

- **Derivation of Metrics from Geometry:** The Minkowski and Schwarzschild metric intervals are revealed not as fundamental postulates of a 4D manifold, but as the Pythagorean identities of the relational carriers S^1 and S^2. Relativistic effects—such as time dilation and length contraction—emerge naturally as geometric rotations between external interaction (Amplitude) and internal existence (Phase).
- **Resolution of Singularities:** By establishing a geometric bound for energy density (ρ_{max}), RG provides a non-singular description of high-energy regions. Gravity becomes "saturated" rather than divergent, effectively resolving the mathematical pathologies found at the centers of black holes without requiring auxiliary quantum corrections.
- **Energetic Closure:** The derivation of the closure law $\kappa^2 = 2\beta^2$ provides a unique geometric origin for the Virial Theorem. This identity proves that the relationship between kinetic and potential energy is dictated by the ratio of their relational degrees of freedom, rather than empirical coincidence.
- **Unification of Projections:** RG eliminates the ambiguity of local energy density by defining WILL as a unified relational resource. What legacy physics treats as separate phenomena—mass, energy, time, and distance—are shown to be locked in a single, self-balancing structure where the ratio ET/ML is identically equal to unity.

While these results offer a computationally simpler and ontologically consistent alternative to coordinate-based physics, empirical alignment remains the ultimate arbiter of any theoretical construct. Relational Geometry does not seek to invalidate the successes of legacy physics but to offer a generative foundation explaining why these laws take the forms they do.

As the first installment of the WILL Trilogy, Part I provides the necessary geometric and algebraic tools to move beyond descriptive modeling. This foundation allows progression to Part II: Relational Cosmology and Part III: Relational Quantum Mechanics, where identical projectional laws apply to the evolution of the Universe and the internal structure of the quantum state. Spacetime is not a container for energy, but the architecture of energy relations themselves—a unified structure defined as WILL.

Spacetime and energy are mutually defining aspects of relational structure we call WILL.

Final Summary

$$\mathbf{SPACETIME} \equiv \mathbf{ENERGY}.$$

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