Observables Only Precession Prediction of S4716 Star within Relational Geometry

Anton Rize[1, *]

[1] *Independent Researcher, WILL Relational Geometry Research*
(Dated: 30.11.2025)

This paper documents a specific numerical prediction made in November 2025, prior to the observational confirmation of the relativistic precession for the star S4716. The short-period S-star S4716 offers a critical laboratory for testing strong-field dynamics around Sagittarius A*. Traditional derivations of its relativistic precession rely on the Schwarzschild metric, dependent on the mass of the central Black Hole ($M_\bullet$) and the gravitational constant G. Here, we present an operational derivation based solely on the *WILL Relational Geometry* framework, which posits a geometric closure condition ($\kappa^2 = 2\beta^2$) between potential and kinematic projections. By de-projecting the line-of-sight velocity from SINFONI 2009 data ($v_{LOS} \approx 1690$ km/s) and utilizing the orbital geometry, we reconstruct the full dynamical state without invoking mass or a spacetime metric or gravitational constant G. We predict a relativistic precession of $\Delta\varphi \approx 14.80$ **arcminutes per orbit**. While numerically equivalent to GR predictions, this derivation demonstrates that the full machinery of curved spacetime geometry may be replaceable by algebraic closure on relational projections. observation of this value by the GRAVITY collaboration would validate the mass-free description of strong-field orbital evolution.

I. INTRODUCTION

The monitoring of stars orbiting the supermassive black hole Sgr A* has provided the most stringent tests of General Relativity (GR) in the strong-field regime. However, standard methods for predicting relativistic effects, such as the Schwarzschild precession, suffer from an epistemic dependency: they require a priori knowledge of the central mass $M_\bullet$ and the gravitational constant G. These are not direct observables; they are derived parameters inferred from the very orbital models one seeks to test.

This raises a fundamental question: Is the concept of "Mass" necessary to predict orbital evolution, or is it merely a historical accounting artifact?

In this Letter, we demonstrate that relativistic orbital dynamics can be derived without reference to mass, the gravitational constant, or the metric tensor. By treating the orbit as a closed geometric system, we employ a *Relational Closure Condition* ($\kappa^2 = 2\beta^2$) - a geometric generalization of the Virial Theorem - to link kinematic intensity directly to potential depth.

We apply this operational framework to the short-period star S4716 [2]. By de-projecting the raw line-of-sight velocity from 2009 SINFONI data, we reconstruct the relativistic state vector and predict a precession of **14.80 arcmin/orbit**. The fact that this result, usually requiring the full machinery of General Relativity, can be derived via algebraic closure suggests that strong-field gravity may be fully described by the algebra of optical observables.

II. RELATIONAL GEOMETRY FORMALISM

In the WILL RG framework, a bound system is defined not by a metric manifold, but by the closure of energy projections onto the minimal relational carriers S^1 (kinematic) and S^2 (potential).

A. The Closure Condition

For any closed system, the partition of the conserved transformation resource between the 1-degree-of-freedom kinematic projection ($\beta = v/c$) and the 2-degree-of-freedom potential projection (κ) is governed by the ratio of their dimensions:

$$\kappa^2 = 2\beta^2. \tag{1}$$

This geometric identity replaces the Virial Theorem. In elliptical systems, this holds for the cycle-averaged projections, while the instantaneous balance is governed by the conservation of the invariant energy W.

B. The Universal Precession Law

The secular evolution of the orbit (precession) arises from the closure defect of the projections at periapsis (p). It is analytically given by:

$$\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2}. \tag{2}$$

Crucially, this formula is algebraically equivalent to the standard GR result ($6\pi GM/c^2 a(1-e^2)$) but is expressed entirely in terms of dimensionless optical scalars.

———————
* egeometricity@gmail.com

III. OPERATIONAL DERIVATION FROM OBSERVATION

To derive the orbital precession of S4716 without relying on the mass of Sgr A* or the Schwarzschild metric, we utilize a scale-invariant reconstruction based on the geometric shape of the orbit (e) and the kinetic intensity (β) derived from raw spectroscopy.

A. Geometric De-projection of Velocity

The observational input is the Line-of-Sight (LOS) velocity measured by SINFONI in 2009 ($v_{LOS} \approx 1690$ km/s) [2]. Since the orbit is highly inclined ($i \approx 161°$), we de-project this value to find the total velocity v_{total}.

The geometric projection factor $\mathcal{P}$ at true anomaly o is:

$$\mathcal{P}(o) = \left| \sin(i) \frac{\cos(\omega + o) + e\cos(\omega)}{\sqrt{1 + e^2 + 2e\cos(o)}} \right|. \quad (3)$$

Using the orbital parameters for S4716 ($e = 0.756$, $i = 161.13°$, $\omega = 2.25°$) and the calculated phase for 2009 ($o \approx 1.122$ rad), we find $\mathcal{P} \approx 0.25$. The total velocity is:

$$v_{total} = \frac{v_{LOS}}{\mathcal{P}} \approx \frac{1690}{0.25} \approx 6760 \text{ km/s}. \quad (4)$$

This yields the scale-invariant intensity parameter at phase o:

$$\beta_o = \frac{v_{total}}{c} \approx 0.02255. \quad (5)$$

B. Propagation to Periapsis

We propagate this state to periapsis using the geometric invariants of WILL.

1. Closure Factor at Observation:

$$\delta_o = \sqrt{\frac{1 + e\cos(o)}{1 + e^2 + 2e\cos(o)}}. \quad (6)$$

2. Potential Projection: From the closure definition, the potential depth κ_o at that phase is:

$$\kappa_o = \beta_o \cdot \delta_o \cdot \sqrt{2}. \quad (7)$$

3. The Energy Invariant (W): The orbital energy invariant W is constant throughout the trajectory:

$$W = \frac{1}{2}\left(\kappa_o^2 - \beta_o^2\right). \quad (8)$$

4. Solution at Periapsis: At periapsis ($o = 0$), the closure factor simplifies to $\delta_p = (1 + e)^{-1/2}$. Solving for the periapsis velocity β_p via invariance of W:

$$\beta_p = \sqrt{\frac{W}{\delta_p^2 - 0.5}}. \quad (9)$$

This yields $\beta_p \approx 0.0265$ ($v_p \approx 7956$ km/s), derived independent of mass models.

IV. PREDICTION

With the derived periapsis intensity $\beta_p \approx 0.0265$ and the corresponding potential $\kappa_p = \beta_p\sqrt{\frac{2}{1+e}}$, we apply the Universal Precession Law (Eq. 2):

$$\Delta\varphi = \frac{3\pi}{2}\frac{\kappa_p^4}{\beta_p^2} \approx \frac{3\pi}{2}(9.13 \times 10^{-4}). \quad (10)$$

$$\boxed{\Delta\varphi \approx 14.7909706972 \approx 14.80 \text{ arcmin/orbit.}} \quad (11)$$

V. DISCUSSION

The result derived here ($\Delta\varphi \approx 14.80'$) matches the expectations of the Schwarzschild metric to high precision. However, the path to this result is physically distinct.

In standard GR, the Virial Theorem ($2K + U = 0$) describes the time-averaged state of the system but does not, by itself, yield the prograde precession. To obtain the $6\pi GM/c^2 a(1 - e^2)$ shift, one must solve the geodesic equations in a curved manifold.

In contrast, the WILL RG treats the factor $\kappa^2 = 2\beta^2$ as a topological constraint on the total energy projections of the closed energy system that includes all channels. It "brething" thru-out elliptical orbits with $\delta \equiv \frac{\kappa^2}{2\beta^2}$ but remain consistent due to the total energy invariant $W = \frac{1}{2}(\kappa^2 - \beta^2)$ = constant at any orbital phase. The fact that a purely algebraic operation on the scalar projections β and κ recovers the exact "curved space" precession implies that the non-linearity attributed to spacetime curvature can be fully accounted for by the non-linearity of the projection geometry (S^1 and S^2).

Furthermore, this method eliminates the "Inverse Problem." We did not need to fit a mass M to the orbit to predict its future. We simply propagated the optical state vector (β_o, δ_o) forward in time using geometric invariance. This represents a significant reduction in ontological complexity.

VI. CONCLUSION

We have derived the relativistic precession of S4716 utilizing only kinematic observables and the geometric closure principle. The result (≈ 14.8 arcmin) is robust against mass uncertainties, as it depends solely on the **algebra of optical observables**.

This prediction serves as a blind test for the WILL Relational Geometry framework. A significant deviation

from 14.8 arcmin/orbit in future GRAVITY measurements would falsify the closure condition $\kappa^2 = 2\beta^2$ in this strong-field application. A confirmation would suggest that the complex machinery of the metric tensor may be subsumed by a simpler, algebraic law of energy conservation.

[1] Gravity Collaboration et al., A&A 636, L5 (2020).

[2] Peißker, F. et al., ApJ 933, 5 (2022).

[3] Rize, A., *WILL Relational Geometry*, Zenodo (2025). DOI: 10.5281/zenodo.17115270.