

WILL Part I

Relational Geometry

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Abstract

This paper, the first in the **WILL Trilogy**:

[Part I: Relational Geometry](#);

[Part II: Relational Cosmology](#);

[Part III: Relational Quantum Geometry](#).

The main goal of this Open Research is to derive the laws of physical reality from the most elementary and geometric principles without introducing any new postulates, axioms, or arbitrary choices. The ideal theory should not rely on pre-existing formalism but naturally arise from the geometry itself.

By applying extreme methodological constraints we established *Relational Geometry (RG)*: a background free foundation where spacetime and energy are sides of the same relational invariant $\text{SPACETIME} \equiv \text{ENERGY}$ that we call WILL. This shift establishes an ontological transition from *descriptive* to *generative* physics: instead of introducing laws to model observations, it derives them as necessary consequences of Relational Geometry itself - turning physics from a catalogue of phenomena into the logical unfolding of inevitable geometrical constrains.

Without metrics, tensors, or free parameters, it reproduces Lorentz factors, the energy-momentum relation, Minkowski and Schwarzschild metric intervals and Einstein field equations via the dimensionless projections β (kinematic) and κ (potential). All known GR critical surfaces (photon sphere, ISCO, horizons) emerge as simple fractions of (κ, β) from the single closure law $\kappa^2 = 2\beta^2$ (topologically derived, virial-like theorem (9.3)).

WILL Part I offers new perspective to several long-standing problems, including:

- **Resolution of GR singularities** (via naturally bounded $\rho_{\max} = \frac{c^2}{8\pi Gr^2}$ 15.2),
- **Derivation of the equivalence principle** (from the common channel of rest-invariant state (8.3) scaling 8.2),
- **Removal of local energy density ambiguity** $\rho = \frac{\kappa^2 c^2}{8\pi Gr^2}$ (14.1),
- **Revelation of a clear relational symmetry between kinematic and potential** projections (??),
- **Establishment of a computationally simpler and ontologically consistent foundation** for subsequent papers on cosmology ([Part II](#)) and quantum mechanics ([Part III](#)).

A Direct Note to the Reader: Why should you read a 60-page independent paper?

The fact that you opened this file and reading these lines already separates you from the majority, and for that, you have my respect. However, sixty pages of unknown research is a substantial commitment, and you should not engage it blindly.

I recommend the following: Be honest with yourself. Define the exact criteria this paper must satisfy to justify the investment of your time. Then, ask [WILL-AI](#) if this work meets your standards. The AI is trained strictly on this document and will give you an honest, unbiased answer based on its contents.

If you are still in doubt, browse [willrg.com](#), interact with the [Galactic Dynamics Lab](#), trace the derivation chain using the [Logos Map](#), review the [falsifiable predictions](#), and have a chat or few with [WILL-AI](#).

Regardless of your decision, I hope you find what you are seeking.

Anton Rize

*This work is archived on Zenodo: ,

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Following yet untamed thought

"There is no such thing as an empty space, i.e., a space without field. . . . Space-time does not claim existence on its own, but only as a structural quality of the field."
— *Albert Einstein, Relativity: The Special and the General Theory* (Appendix V: "Relativity and the Problem of Space"), 1952 edition, Methuen (London), p. 155; based on earlier 1920 additions.

IMPORTANT:

This document must be read **literally**. All terms are defined within the relational logic of WILL Relational Geometry.

Definition of Geometry: The term "geometry" is used herein in its fundamental, classical sense: the pure logic of relations, ratios, and structural constructions. It **does not** refer to modern differential geometry. There is no *a priori* metric tensor ($g_{\mu\nu}$), no Riemannian manifold, and no underlying coordinate grid. The geometry emerges from physical relations, not analytically from a background space.

Any attempt to reinterpret these derivations through conventional metric notions (*absolute distances, external backgrounds, hidden containers*) will produce distortions and mathematical misreadings.

Take the words as written and ontology as derived, without smuggling external concepts.

1 Foundational Methodological Principles

Every logical step is subject to empirical audit: if any consequence of this chain contradicts observation, the chain fails.

As humans, we often project ourselves onto observed phenomena, obscuring the true nature of reality behind our anthropic tendencies. In order to prevent such misleading practices, we must establish extreme methodological constraints. These constraints are not statements about the nature of reality, but the strict rules of logical purity for constructing a theory. Without strict methodology, any theoretical construct risks that excessive freedom of interpretation will result in a framework that tells us more about ourselves than about the nature of the Universe.

The philosophical reasoning behind these particular methodological constraints might deserve its own dedicated publication and can be challenged, but this research's main domain lies within physical reality that can be empirically tested; therefore, empirical alignment is the ultimate test of the methods presented.

Flow chart of derivation logic

[Logos Map](#)

Principle 1.1 (Epistemic Hygiene). Epistemic Hygiene as Refusal to Import Unjustified Assumptions. *This line of reasoning derive physics by **removing hidden assumptions**, rather than introducing new postulates. This construction is deliberate and contains zero free parameters. This is not a simplification - it is a deliberate epistemic constraint. No assumptions are introduced and no constructs are retained unless they are geometrically or energetically necessary.*

Principle 1.2 (Ontological Minimalism). *Any fundamental theory must proceed from the minimum possible number of ontological assumptions. The burden of proof lies with any assertion that introduces additional complexity or new entities.*

Principle 1.3 (Relational Origin). All physical quantities must be defined by their relations. *Any introduction of absolute properties risks reintroducing metaphysical artefacts and contradicts the foundational insight of relationalism.*

Principle 1.4 (Simplicity). **Everything** must be expressed in the **simplest** form possible. *Any unjustified complexity risks reintroducing metaphysical artefacts and contradicts the foundational insight of Epistemic Hygiene.*

Principle 1.5 (Mathematical Transparency). 1. *Every mathematical phrase, operational choice, or identity carries its own ontological statement.*

2. *Each mathematical object must correspond to explicitly identifiable relation between observers with transparent ontological origin.*

3. *Every symbol must be anchored to unique physical idea.*

4. *Introducing symbols without explicit necessity constitutes **Over-parameterization**: the proliferation of symbols without corresponding physical meaning.*

5. *Number of symbols = Number of independent physical ideas.*

Mathematics is a language, not a world. Its symbols must never outnumber the physical meanings they encode.

1.1 The Axiomatic Core: Relational Geometry

This foundational methodological principles lead us to the axiomatic core of relational geometry:

Theorem 1.6 (Topological Closure). *A purely relational system cannot be physically open; it is closed by absolute analytic necessity.*

Proof by Analytic Tautology. 1. **Premise 1 (Relational Origin):** Within RG Physical quantities are defined exclusively by their relations (Principle 1.3).

2. **Premise 2 (Interaction as Integration):** If a hypothetical “outside” entity or boundary were to physically interact with the system, that interaction itself constitutes a relation.

3. **Deduction (The Tautology):** As relation is formed, it is mathematically and physically integrated into the system.

An “open relational system” experiencing external influence is a logical contradiction. The system is closed not by a spatial boundary or a metric container, but by the strict impossibility of an interacting non-relation. The system is topologically closed by definition. □

1.2 What is Energy in a Relational Framework?

Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, chemical — yet all are interchangeable, pointing to a single underlying structure.

Crucially, this quantity is never observed directly, but only through *differences between states*: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation.

Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself.

Theorem 1.7 (Relational Invariance). *Within a purely relational architecture devoid of an external background, the continuity of change strictly necessitates the existence of conserved invariant measure of change.*

Proof by Structural Necessity. 1. **Premise 1 (Relational Origin):** Physical quantities are defined exclusively by their relations (Principle 1.3).

2. **Premise 2 (Empirical Change):** States undergo observable change.

3. **Premise 3 (Topological Closure):** As proven in Theorem 1.6, the relational system is analytically closed. There is no external background, void, or hidden container to absorb or inject influence.

4. **Deduction (Causal Continuity):** Because the system is closed (Premise 3), an empirical change (Premise 2) cannot vanish into a void or emerge from one. To maintain the structural integrity of the relations (Premise 1), any change must be perfectly balanced by a complementary change elsewhere in the system.

Therefore, within this strictly closed relational network, there must exist a continuous, invariant quantitative measure of change. □

Corollary 1.8 (Historical Note on Energy). *In classical and relativistic mechanics, this derived invariant measure of change is assigned the historical label “Energy”. By proving this invariance geometrically, we strip “Energy” of its substantialist properties (as a “substance” or “thing”) and reveal its origin as the necessary bookkeeping of causality.*

Energy :

Definition 1.9 (Energy).

*Energy is the invariant relational measure
of state transformation
within a topologically closed system.*

Theorem 1.10 (Relational Isotropy). *Within a background-free relational system, no spatial direction or coordinate can be a priori privileged. Therefore, the fundamental geometry encoding the conserved relational resource (energy) must be maximally symmetric (isotropic).*

Proof by Relational Origin. 1. **Premise 1 (Absence of a Grid):** As established by Theorem 1.6 (Topological Closure), the relational system possesses no external container, absolute metric tensor, or underlying coordinate grid.

2. **Premise 2 (Direction requires Relation):** Under the Principle of Relational Origin (Principle 1.3), “direction” or “orientation” cannot exist in a void; it only physically manifests as a specific interaction or measurement between defined participants.

3. **Deduction (Pure Gauge):** Because there is no absolute grid (Premise 1), assigning an intrinsic, absolute axis to the unmeasured relational resource (energy) introduces unobservable “surplus ontology.” Any such arbitrary orientation is a mathematical artifact (pure gauge).

Therefore, prior to a specific interaction defining a local axis, the pure relational structure encoding this conserved resource must possess no intrinsic privileged direction. It must be strictly isotropic. \square

2 The Last Geocentric Epicycle

The standard formulation of General Relativity often relies on the concept of a pre-existing, container-like spacetime that then gets “filled” with fields and matter. This is in direct tension with the Relational Principle 1.3.

The standard derivation of the Einstein field equations begins with the Einstein-Hilbert action, which is built upon the metric as the fundamental variable. This metric is defined on a smooth manifold that is assumed to exist *a priori*. This manifold, even without a metric, carries topological and differential structure - an absolute scaffold. This constitutes surplus ontology. It violates Principle 1.2 (Ontological Minimalism) because it introduces an entity (the manifold) that is not derived from relational observations.

In the standard formulation, the stress-energy tensor is derived from the variation of the matter Lagrangian with respect to the metric. This assumes that energy is a property of matter fields that can be localized in spacetime. However, this localization is frame-dependent (via the equivalence principle) and leads to well-known problems such as the non-uniqueness of the gravitational energy-momentum pseudotensor. This is a direct violation of the Relational Origin principle 1.3: energy is treated as an absolute property of matter rather than a relational measure.

We proceed from strict epistemic minimalism, disallowing all background structures, even hidden or asymptotic ones.

2.1 Historical Pattern: breakthroughs delete, not add

- **Copernicus** eliminated the Earth/cosmos separation.
- **Newton** eliminated the terrestrial/celestial law separation.
- **Maxwell** eliminated the electricity/magnetism separation.
- **Einstein** eliminated the space/time separation.

Each step widened the relational circle and reduced the number of primitives - an unexplained absolutes. The spacetime–energy split is the only survivor of this pruning sequence.

2.2 The contemporary split: an unjustified ontological commitment

All present-day theories (SR, GR, QFT, CDM, Standard Model) are built with a *bi-variable* syntax:

$$\underbrace{\text{fixed manifold} + \text{metric}}_{\text{structure}} + \underbrace{\text{fields} + \text{constants}}_{\text{dynamics}}.$$

No observation demands this duplication; it is retained purely because the resulting Lagrangians are empirically adequate *inside* the split. The split is therefore *not* an empirical discovery but an unjustified ontological statement.

2.3 Empirical deficit of the separation

- **Local energy conservation** verified only *after* the metric is declared fixed; no experiment varies the *volume* of flat space and checks calorimetry.
- **Universality of free fall** tests $m_i = m_g$ numerically, not the claim that inertia resides *in* the object rather than in a geometric scaling relation.
- **Gravitational-wave polarisations** test spin content, not ontology; extra modes can still be called “matter on spacetime”.

- **Casimir/Lamb shifts** measure *differences* of vacuum energy between two geometries; the absolute bulk term is explicitly subtracted, leaving the split intact.

Every “test” is an *internal consistency check* of a formalism that already presupposes two substances. None constitute *positive evidence* for the split.

2.4 Consequence

Until an experiment varies the amount of space while holding everything else fixed, the spacetime–energy separation remains an *unevidenced metaphysical postulate*. **It is the last geocentric epicycle in physics.**

Summary

Any attempt to treat “*spacetime structure*” as separate from “*dynamics*” smuggles in a background container that is not justified by the phenomena. This violates epistemic hygiene: it introduces an ontological artifact without necessity. Eliminating this separation compels the identification of structure and dynamics as two aspects of a single entity.

2.5 The Unifying Principle: Removing the Hidden Assumption

Lemma 2.1 (False Separation). *Any model that treats processes as unfolding within an independent background necessarily assigns to that background structural features (metric, orientation, or frame) not derivable from the relations among the processes themselves. Such a background constitutes an extraneous absolute.*

Proof. Suppose an independent background exists. Then at least one of its structural attributes - metric relations, a preferred orientation, or a class of inertial frames - remains fixed regardless of inter-process data. This attribute is not relationally inferred but posited a priori. It thereby violates the relational closure principle: it introduces a non-relational absolute external to the system. Hence the separation is illicit. \square

Corollary 2.2 (Structure–Dynamics Coincidence). *To avoid the artifact of Lemma 2.1, the structural arena and the dynamical content must be identified: geometry is energy, and energy is geometry.*

Principle 2.3 (Unifying Ontological Principle: Removing the Hidden Assumption).

$$\boxed{SPACETIME \equiv ENERGY}$$

This is not introduced as a new ontological entity but as a Principle with negative ontological weight: it removes the hidden unjustified separation between "structure" and "dynamics." Spacetime is not a "container" but fundamental relational energy structure.

Remark 2.4 (Auditability). *Principle 2.3 is foundational but testable: it is subject to (i) geometric audit (internal logical consequences) and (ii) empirical audit (agreement with empirical data).*

Definition 2.5 (WILL). **WILL \equiv SPACE-TIME-ENERGY** *is the technical term we use for the unified relational structure determined by Unifying Principle refpr:unifying. All physically meaningful quantities are relational features of WILL; no external container is permitted.*

Summary:

This Principle does not add, it subtracts: it removes the hidden assumption. Structure and dynamics are two aspects of a single entity that we call WILL.

3 WILL Structure

Having established Principle 2.3 by removing the illicit separation of structure and dynamics, we now proceed to derive its necessary geometric and physical consequences.

The axiomatic core (1.1)—Closure, Conservation, and Isotropy—establishes the necessary properties of the geometric carriers of the relational resource (energy).

Relational Carrier Conventions:

All references to “carriers” within WILL RG open research are to be read in the strict relational sense:

- **Degree of freedom (DOF):** An n -dimensional relational carrier encoding the conserved transformation resource.
- **Direction:** An oriented relation. Opposite orientations are physically distinct and not identified.
- **Closed carrier:** Compact and without boundary; resource cannot leak into an external reservoir.
- **No background:** No external embedding space. All geometric structure is reconstructible solely from relations between participants.
- **Maximal symmetry:** The carrier is homogeneous and isotropic with respect to oriented directions.
- **Minimal relational carrier:** A closed, maximally symmetric carrier utilizing exactly the required DOF without arbitrary parameters.

Deriving Relational Carriers

Theorem 3.1 (Minimal Relational Carriers of the Conserved Energy Resource). *The minimal relational carriers satisfying Closure, Conservation, Maximal Symmetry, and Mathematical Transparency are:*

- (a) S^1 for directional (Kinematic) relational transformation;
- (b) S^2 for omnidirectional (Potential) relational transformation.

Proof. The proof classifies the minimal types of relations and applies the derived geometric constraints:

- (a) **1-DOF (Kinematic) Relation:** The minimal non-trivial relation involves 1 degree of freedom (1-DOF): a sequence of state transformations.

By Theorem 1.6 (Closure), this sequence must form a loop to prevent resource leakage. A discrete sequence (a finite cyclic group C_n) requires assigning a specific integer value n to the number of states. Principle 1.5 (Mathematical Transparency) prohibits the introduction of arbitrary numerical parameters without unique physical justification. To achieve a parameter-free structure, the state cardinality must remain undefined, forcing the continuum limit.

By Theorem 1.10 (Isotropy), this continuous 1-DOF loop must be maximally symmetric. The unique connected, closed, parameter-free 1-carrier is the continuous S^1 (circle).

- (b) **2-DOF (Potential) Relation:** The next minimal relation involves 2 degrees of freedom (2-DOF): the omnidirectional distribution of the transformational resource from a primary state.

This requires a 2-dimensional carrier. By Principle 1.5, any discrete tiling of this distribution introduces arbitrary parameters (such as the number of nodes or faces), strictly forcing a continuous 2-carrier.

By Theorem 1.6, this continuous 2-carrier must be closed. By Theorem 1.10, it must be maximally symmetric (an isotropic distribution of potential).

Topological classification of a closed 2-carrier under the strict requirement of maximal continuous symmetry eliminates all anisotropic carriers (e.g., the torus T^2 , which possesses preferred intrinsic axes). The unique parameter-free, maximally symmetric 2-carrier is S^2 (the 2-sphere).

Ontological Minimalism and Mathematical Transparency uniquely enforce S^1 and S^2 as the minimal relational carriers without invoking metric geometry *a priori*. □

Remark 3.2 (Non-spatial Reading). S^1 and S^2 are not to be interpreted as spacetime geometries. They are relational carriers encoding the closure, conservation, and isotropy of the transformational resource. Spatial and temporal notions are emergent descriptors of patterns within WILL.

Remark 3.3 (Constructive Derivation Protocol). *This derivation proceeds by construction from traced origins. Each object entering the chain - the principles, the theorems, the carriers - has an explicit derivation path. Alternative structures (discrete groups, higher-dimensional carriers, non-orientable surfaces) are not excluded by enumeration; they are absent because no derivation chain within this framework produces them. The burden of justification lies with any proposed addition, not with the derived structure.*

Methodological Constraints
↓ (<i>apply to existing physics</i>)
False Separation Identified
↓ (<i>dissolve</i>)
SPACETIME ≡ ENERGY
↓ (<i>derive consequences</i>)
Closure + Conservation + Isotropy
↓ (<i>classify carriers</i>)
S^1 (1-DOF kinematic) + S^2 (2-DOF potential)

3.1 The Amplitude-Phase Duality

The manifestation of any system is distributed between its internal (Phase) and external (Amplitude) aspects. This single geometric constraint gives rise to the core phenomena of modern physics:

Lemma 3.4 (Duality of Evolution). *The identification of spacetime with energy and its transformations necessitates two complementary relational measures:*

1. the **amplitude** of transformation (*external shift*), and
2. the **phase** of transformation (*internal order*).

Proof. Any complete description of transformation must specify both what changes and how that change is internally ordered. A single measure cannot capture both. The Relational Carriers S^1 and S^2 provide the minimal geometry enforcing such complementarity: their orthogonal projections furnish precisely two non-redundant, coupled coordinates. □

The orthogonal decomposition of the relational carriers S^1 and S^2 reveals a functional duality. Physical state is a combination of two projections:

Definition 3.5 (The Amplitude Projection (External Interaction)). *Denoted by β (kinematic) and κ (potential). This component measures the extent of external relation. It manifests as momentum or potential intensity. Physically, it represents the system's relational "shift" from the **Observer's Relational Origin** (the rest frame).*

$$\text{Amplitude} \rightarrow \text{External Power (Kinetic/Potential)}$$

Definition 3.6 (The Phase Projection (Internal Evolution)). *Denoted by β_Y and κ_X . This component measures the internal ordering. It governs the intrinsic scale of proper time and proper length. A Phase of 1 represents maximal internal flow (rest), while a Phase of 0 represents the collapse of internal causality (light-speed/event horizon).*

$$\text{Phase} \rightarrow \text{Internal Order (Time/Structure)}$$

Theorem 3.7 (Universal Conservation of Relation). *For both kinematic (S^1) and potential (S^2) modes, the sum of the squared Amplitude and squared Phase is strictly invariant:*

$$\underbrace{\text{Amplitude}^2}_{\text{External Interaction}} + \underbrace{\text{Phase}^2}_{\text{Internal Existence}} = 1 \tag{1}$$

Proof. By Theorem 3.1, the conserved relational resource is carried by the maximally symmetric geometries S^1 and S^2 . Normalizing the total conserved resource to unity (1), any physical state corresponds to a point on these carriers. The algebraic identity of orthogonal projections on a unit circle and sphere strictly dictates that the sum of the squares of the coordinates must equal the squared radius. Therefore, the parameters satisfy $\beta^2 + \beta_Y^2 = 1$ and $\kappa^2 + \kappa_X^2 = 1$, encoding the finite, closed relational budget. □

3.2 Consequence: Relativistic Effects

Proposition 3.8 (Physical Interpretation: Relativistic Effects). *The conservation law of Theorem 3.7 implies that any redistribution between the orthogonal components manifests physically as the relativistic effects of time dilation and length contraction.*

Proof. By Theorem 3.7, the components satisfy $\beta^2 + \beta_Y^2 = 1$ and $\kappa^2 + \kappa_X^2 = 1$. An increase in Amplitude (β, κ) strictly enforces a geometric decrease in Phase measure (β_Y, κ_X). This necessary reduction of β_Y and κ_X corresponds precisely to the dilation of proper time and the contraction of proper length, while the growth of β and κ represents momentum or potential depth. Thus, the kinematic and potential trade-off is the direct, un-parameterized physical expression of the geometric closure of the S^1 and S^2 carriers. \square

Summary:

The geometry of spacetime is dictated by the Relational Geometry of energy states.

Remark 3.9. *This identifies Relativistic and Gravitational Time Dilation not as a mysterious “slowing down” of clocks, but as a strict geometric **phase rotation**. As a system invests more of its relational existence into external Amplitude (β or κ), it necessarily withdraws from its internal Phase (β_Y or κ_X), changing the rate of its proper time evolution.*

3.3 Energy as a Relation - What κ and β Actually Mean

Energy (Definition 1.9) is the measure of differences between states.

In the relational framework:

- Physical parameters like energy, speed, and gravitational potential do not belong to objects.
- Instead, they represent how we, as observers, measure an object’s state differences from our own.

In this view, your perspective is always the reference frame. You are always at the point of origin (0,0) on your (β, κ) plane. At this origin, your internal phases are maximal ($\beta_Y = 1, \kappa_X = 1$), ensuring the conservation equations hold for the observer’s self-state.

Everything else is described by how it differs from your state:

- β is the measure of how much of the universal “transformation rate” you see as motion through space, relative to yourself.
- κ is the measure of how deeply an object sits in a potential well, as seen from your position. It is your personal “ruler” for potential depth.

Think of κ and β as your own relational measuring tools:

- β is how far along your “kinetic ruler” you project another object’s state.
- κ is how deep into your “potential well” you see another object’s state.

Thus, relational understanding follows naturally:

- Energy is the capacity to move between states.
- Saying “the object’s energy” always implicitly means “the object’s energy as measured from your perspective.”

Here is a simple classical analogy:

Imagine standing on a train platform. A train passes by rapidly: to you, it has significant kinetic energy. But if you jump onto the train, it instantly becomes stationary relative to you. Its kinetic energy is now zero — because your frame of reference shifted. The energy did not vanish; your relational perspective changed.

Summary:

- The projections κ and β are your personal, relational measurements of state difference.
- All measurement boils down to describing how things differ in relation to the observer.

3.4 Ontological Status of the Relational Carriers S^1 and S^2

A natural question arises regarding the ontological status of the circle S^1 and the sphere S^2 : What are they, and where do they “exist”?

The answer requires a shift in perspective. In WILL Relational Geometry, S^1 and S^2 are **not spatial entities** existing within a pre-defined container. They are the necessary **relational architectures** that implement the core identity $\text{SPACETIME} \equiv \text{ENERGY}$.

Energy as Relational Transformation Capacity Recall that energy is defined as the *relational measure of difference between possible states*. It is not an intrinsic property but a relational potential for change. It is never observed directly, only through transformations.

The Carriers as Protocols of Interaction The Carriers S^1 and S^2 are the minimal, unique mathematical structures capable of hosting this relational “bookkeeping” for directional and omnidirectional transformations, respectively. They enforce closure, conservation, and symmetry by their very topology.

Imagine two observers, A and B :

- Observer A is the center of their own relational framework. Observer B is a point on A 's S^1 (for kinematic relations) and S^2 (for gravitational relations).
- Simultaneously, observer B is the center of their own framework. Observer A is a point on B 's S^1 and S^2 .

There is no privileged “master” carrier. Each observable interaction is structured by these mutually-centered relational protocols. The parameters β and κ are the coordinates within these relational dimensions, and the conservation laws (e.g., $\beta^2 + \beta_Y^2 = 1$; $\kappa^2 + \kappa_X^2 = 1$) are the innate accounting rules of these protocols.

3.5 Manifestation of Spacetime

In this construction, “space” and “time” are not treated as separate, fundamental aspects of reality. Instead, they are shown to manifest as necessary physical consequences of a single, underlying principle: the geometry of a closed, relational system.

Therefore, the question “Where are S^1 and S^2 ?” is a category error. They are not *in* space; they are the structures whose coordinated, multi-centered interactions **give rise to** the phenomenon we perceive as spacetime. Spacetime is the geometric “shadow” cast by the dynamics of energy relations projected onto these architectures.

In essence, S^1 and S^2 are the ontological embodiment of the relational principle. They are derived as structures that can house the transformational resource (energy) in a closed, conserved, and isotropic system. Their status is that of a fundamental **Relational Geometry** from which physics is generated.

4 Kinetic Energy Projection on S^1

Since S^1 encodes one-dimensional shift, the total energy E of the system must project consistently onto both axes:

$$E^2 = E_\beta^2 + E_{\beta_Y}^2$$

where

$$E_X = E\beta, \quad E_Y = E\beta_Y.$$

Desmos project

[Kinetic projection \$\beta\$ on the \$S^1\$ carrier \(Special Relativity\)](#)

4.1 Geometric Nature of Rest Energy State

To derive the energy transformation, we must first explicitly identify the physical meaning of the orthogonal projections.

Theorem 4.1 (Invariant Projection of Rest Energy). *For any state (β, β_Y) on the relational circle S^1 , the total energy E must scale such that its vertical projection remains constant and equal to the rest energy E_0 .*

$$E\beta_Y = E_0.$$

Proof. Let the total energy E be distributed on the relational carrier S^1 with projections β (kinematic amplitude) and β_Y (internal phase).

$$E_Y = E \cdot \beta_Y$$

By Principle 1.3 (Relational Origin), the internal structure of an object (its rest energy E_0) is defined solely by relations internal to itself. Therefore, it must be invariant under changes in its relation to an external observer. In relation to itself kinematic projection is always ($\beta = 0 \implies \beta_Y = 1$). Therefore:

$$E_Y \equiv E_0$$

where E_0 is the energy measured in the frame where the system is at rest ($\beta = 0, \beta_Y = 1$). Thus, geometric consistency requires the vertical leg to be fixed:

$$E\beta_Y = E_0 \implies E = \frac{E_0}{\beta_Y}.$$

The "hypotenuse" (Total Energy E) is therefore not a fixed-length vector that rotates (which would reduce E_Y), but a scalable relational magnitude that grows to preserve the invariant vertical leg E_0 against the closure constraint $\beta_Y = \sqrt{1 - \beta^2}$. \square

Summary:

The historical Lorentz factor γ is the reciprocal of β_Y . $\gamma = 1/\beta_Y$

4.2 The Geometric and Historical Nature of Mass

Corollary 4.2 (Rest Energy and Mass Equivalence). *This geometry does not require kilograms as units or mass as a concept. Within the normalization $c = 1$, the invariant rest energy E_0 can be expressed as human-convention-driven parameter of mass in kilograms:*

$$E_0 = m.$$

Proof. From the invariant projection $E\beta_Y = E_0$ and closure of S^1 , no additional scaling parameter is required. Hence the conventional bookkeeping identities $E_0 = mc^2$ or $m = E_0/c^2$ reduce to tautologies. Mass is therefore not independent, but the rest-energy invariant itself. \square

Summary:

Mass is the invariant projection of total rest energy expressed in anthropocentric units of kilograms.

4.3 Energy–Momentum Relation

Proposition 4.3 (Horizontal Projection as Momentum). *On the relational circle, the unique linear geometric projection measure of external amplitude from rest is the horizontal projection $E_X = E\beta$; hence*

$$p \equiv E_X \equiv E\beta \quad (c = 1).$$

Proof. The rest state is $(\beta, \beta_Y) = (0, 1)$. A shift measure must (i) vanish at rest, (ii) grow monotonically with $|\beta|$, and (iii) flip sign under $\beta \mapsto -\beta$. The only relational candidate satisfying (i)-(iii) is the horizontal projection $E_X = E\beta$. Thus the identification is necessary and unique within methodological constraints. \square

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[Energy-momentum Triangle](#)

Corollary 4.4 (Energy–Momentum Relation). *With p identified by Proposition 4.3 and $m = E_0$, the closure identity yields*

$$E^2 = p^2 + m^2 \quad (c = 1).$$

Equivalently, upon restoring c ,

$$E^2 = (pc)^2 + (mc^2)^2.$$

Proof. By closure, $(E\beta)^2 + (E\beta_Y)^2 = E^2$. Substituting $p = E\beta$ and $m = E_0$ proves the claim. Restoring c is dimensional bookkeeping: $p \mapsto pc$ and $m \mapsto mc^2$, while E remains E , yielding the standard form. \square

Remark 4.5 (Geometric Forms). *The same identity may be expressed explicitly in terms of circle coordinates:*

$$E^2 = \left(\frac{\beta}{\beta_Y} E_0\right)^2 + E_0^2 = (\cot(\theta_1) E_0)^2 + E_0^2.$$

These are equivalent renderings of the same geometric necessity.

Remark 4.6 (Units sanity check - bookkeeping). *Using $\beta = v/c$, the identification $p \equiv E\beta$ gives*

$$pc = E \frac{v}{c} \implies p = \frac{E v}{c^2}.$$

With $E = \frac{1}{\beta_Y} mc^2 = \gamma mc^2$ this reduces to $p = \frac{\beta}{\beta_Y} mc = \gamma mv$, the standard relativistic momentum. No new parameters are introduced.

$\beta = v/c \quad \theta_1 = \arccos(\beta)$	
Algebraic Form	Trigonometric Form
$\beta = v/c = \sqrt{1 - \beta_Y^2}$	$\beta = \cos(\theta_1)$
$\beta_Y = \sqrt{1 - (v/c)^2} = \sqrt{1 - \beta^2}$	$\beta_Y = \sin(\theta_1)$

Table 1: Geometric representation of relativistic effects.

Summary

The energy–momentum relation $E^2 = (pc)^2 + (mc^2)^2$ is a geometric identity of S^1 carrier.

4.4 Kinematic Projection Equivalence: The Minkowski metric interval as an S^1 Pythagorean Identity

The Minkowski spacetime interval, posited by legacy Special Relativity as a physical metric of a 4D background manifold, mathematically collapses into the invariant phase budget of the kinematic relational carrier (S^1).

The standard interval formulation relies on absolute spatial and temporal coordinates:

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \quad (2)$$

Restricting relative translation to a single spatial degree of freedom (dx) eliminates orthogonal components while maintaining exact algebraic equality:

$$c^2 d\tau^2 = c^2 dt^2 - dx^2 \quad (3)$$

Dividing the relation by the coordinate phase component ($c^2 dt^2$) extracts the dimensionless relational ratios:

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \left(\frac{1}{c} \frac{dx}{dt}\right)^2 \quad (4)$$

Substituting the standard kinematic definition of velocity ($v = dx/dt$) and rearranging the terms produces the constraint of the WILL Relational Geometry:

$$\left(\frac{v}{c}\right)^2 + \left(\frac{d\tau}{dt}\right)^2 = \beta^2 + \beta_Y^2 = 1 \quad (5)$$

This derivation mandates the explicit legacy translation mapping:

- **Kinematic Projection (β):** $\beta \equiv \cos(\theta_1) = \frac{v}{c}$
- **Internal Phase Projection (β_Y):** $\beta_Y \equiv \sin(\theta_1) = \sqrt{1 - \beta^2} = \frac{d\tau}{dt} = \frac{1}{\gamma}$

Epistemological Consequence: Elimination of the Manifold

The Lorentz factor (γ) is mathematically identical to the reciprocal of the internal phase projection $\gamma = \frac{1}{\beta_Y} = \frac{1}{\sin \theta_1}$. Kinematic time dilation ($\beta_Y < 1$) is not a physical deformation of a spatial fabric, but a strict geometric rotation on the S^1 unit circle. The constraint $\beta^2 + \beta_Y^2 = 1$ fully replicates the invariant scalar properties of Minkowski space without requiring a background spatial coordinate system.

5 Potential Energy Projection on S^2

IMPORTANT:

Throughout this work, S^1 and S^2 are not to be interpreted as spacetime geometries but purely as relational carriers encoding energy conservation. Any reading otherwise is a misinterpretation.

Analogous to S^1 , the relational geometry of the sphere, S^2 , provides orthogonal projections for two aspects of omnidirectional transformation. We define them as follows:

- **The Amplitude Component (κ):** This projection represents the *relational potential measure* between the object and the observer. It corresponds to the *extent* of transformation, which manifests physically as potential depth. A value of $\kappa = 1$ denotes *saturation*: the entire relational resource of the system has been allocated into the potential channel. No residual capacity remains for kinematic phase progression. This condition mathematically defines the relational horizon (event horizon).
- **The Phase Component (κ_X):** This projection governs the intrinsic scale of proper length and proper time units within the potential field, corresponding to the internal *sequence* of transformation.

These two components are not independent but are bound by the fundamental conservation law of the closed system, which acts as a finite “budget of transformation”:

$$\kappa_X^2 + \kappa^2 = 1$$

The manifestation of any system is distributed between its internal (Phase) and relational (Amplitude) aspects. This single geometric constraint gives rise to the core gravitational phenomena of modern physics.

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Potential projection as κ on S^2 carrier

5.1 Potential Meridional Section of S^2

By isotropy, the omnidirectional carrier is S^2 , but any radially symmetric exchange reduces to a great-circle meridional section. We therefore work on a unit great circle of S^2 with the parameterization $(\kappa_X, \kappa) = (\cos \theta_2, \sin \theta_2)$.

5.2 Consequence: Gravitational Effects

The redistribution of the relational budget between the Phase and Amplitude components directly produces the effects of General Relativity. An increase in the relational amplitude measure (κ , gravitational potential) strictly necessitates a geometric decrease in the measure of the internal structure (κ_X). This geometric trade-off is observed physically as gravitational length contraction and time dilation. Thus, the curved geometry of spacetime is merely the shadow cast by the geometry of conserved relations.

5.3 Potential Equivalence: The Schwarzschild Metric Interval as an S^2 Pythagorean Identity

This derivation demonstrates how the Schwarzschild metric interval, a cornerstone of General Relativity (GR), algebraically maps directly to the closure identity of the potential carrier, $\kappa^2 + \kappa_X^2 = 1$, within WILL Relational Geometry (RG). This is strictly analogous to how the Minkowski interval maps to the kinematic closure identity $\beta^2 + \beta_Y^2 = 1$.

The Schwarzschild metric describes the spacetime geometry around a spherically symmetric, non-rotating mass M . For a static observer (i.e., not moving radially or angularly, so $dr = 0$, $d\theta = 0$, $d\phi = 0$), the metric interval is:

$$ds^2 = c^2 \left(1 - \frac{R_s}{r} \right) dt^2$$

where $R_s = \frac{2GM}{c^2}$ is the Schwarzschild radius.

The proper time $d\tau$ experienced by the static observer is related to the metric interval by $ds^2 = c^2 d\tau^2$. Substituting this into the Schwarzschild interval yields:

$$c^2 d\tau^2 = c^2 \left(1 - \frac{R_s}{r} \right) dt^2$$

Dividing both sides by the coordinate phase component $c^2 dt^2$, we extract the dimensionless relational ratio (the squared gravitational time dilation factor):

$$\left(\frac{d\tau}{dt} \right)^2 = 1 - \frac{R_s}{r}$$

In WILL Relational Geometry, the potential phase component $\kappa_X \equiv \frac{d\tau}{dt}$ governs the intrinsic scale of proper time in a gravitational field. Furthermore, the pure relational potential amplitude is defined as $\kappa^2 \equiv \frac{R_s}{r}$.

Note on Epistemic Hygiene: In sections (??, ??, ??) we will prove that the classical gravitational constant (G) embedded in R_s is not a foundational primitive of reality, but a historical human bookkeeping constant required to translate the pure, dimensionless geometric ratio (κ^2) into arbitrary metric units (meters, kilograms).

Substituting these pure relational identities back into the ratio, we arrive at the fundamental closure identity for the S^2 potential carrier:

$$\frac{R_s}{r} + \left(\frac{d\tau}{dt}\right)^2 = \kappa^2 + \kappa_X^2 = 1$$

This derivation mandates the explicit legacy translation mapping:

- **Potential Amplitude Projection (κ):** $\kappa \equiv \sin(\theta_2) = \sqrt{\frac{R_s}{r}}$
- **Internal Phase Projection (κ_X):** $\kappa_X \equiv \cos(\theta_2) = \sqrt{1 - \kappa^2} = \frac{d\tau}{dt}$

5.4 Epistemological Consequence: Unveiling the Geometric Structure

This exercise demonstrates that the algebraic structure of the Schwarzschild metric's g_{tt} component directly maps to the closure identity of the S^2 potential carrier in WILL RG. The terms in GR's description of spacetime curvature (specifically, the gravitational time dilation factor) are shown to be directly equivalent to the squared potential phase and amplitude projections of WILL RG.

Key Insight

While the Schwarzschild metric *describes* gravitational effects within a pre-existing spacetime manifold, WILL RG *generates* the underlying relational geometry ($\kappa^2 + \kappa_X^2 = 1$) from its foundational ontological principle (**SPACETIME** \equiv **ENERGY**). This algebraic correspondence indicates that the physical reality captured by GR is a projected phenomenon entirely consistent with the more fundamental relational structure of WILL RG. The geometric identity $\kappa^2 + \kappa_X^2 = 1$ is thus not merely a re-parameterization, but a direct consequence of the removal of hidden assumptions about the separation of structure and dynamics.

5.5 Gravitational Tangent Formulation

Just as the relativistic energy–momentum relation can be expressed in terms of the kinematic projection $\beta = v/c$, we may construct its gravitational analogue using the potential projection $\kappa = v_e/c$, where v_e is the Newtonian escape velocity at radius r .

In the kinematic case, with $\beta = \cos \theta_1$, the energy relation can be written as

$$E^2 = (\cot \theta_1 E_0)^2 + E_0^2, \tag{6}$$

so that the relativistic momentum is expressed as

$$p = (E_0/c) \cot \theta_1. \tag{7}$$

In full geometric symmetry, the gravitational case follows from $\kappa = \sin \theta_2$. We define the scalable gravitational total energy as

$$E_g = \frac{E_0}{\kappa_X}, \quad \kappa_X = \sqrt{1 - \kappa^2}, \tag{8}$$

and introduce the gravitational analogue of momentum (potential momentum):

$$p_g = (E_0/c) \tan \theta_2. \tag{9}$$

This yields the exact gravitational energy–momentum relation:

$$E_g^2 = (p_g c)^2 + (m c^2)^2. \tag{10}$$

Summary:

$$\begin{aligned} \beta &= \cos \theta_1, & \kappa &= \sin \theta_2, \\ \beta &\longleftrightarrow \kappa, & \cot \theta_1 &\longleftrightarrow \tan \theta_2. \end{aligned}$$

Kinematic momentum p and gravitational momentum p_g are thus dual projections of the closed relational geometry, expressed through complementary trigonometric forms.

6 Total Relational Shift Q

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Q as Total Relational Shift

When an observer observes another system, they assign to it a Total Relational Shift norm Q :

$$Q^2 = \beta^2 + \kappa^2 \quad (11)$$

Relational reciprocity is the invariance of this norm under the self-centering operation of each observer.

Each observer places itself at the relational origin

$$(\beta, \kappa) = (0, 0).$$

If the other system now looks back, it again self-centres at $(0, 0)$ and applies the same rule. It measures the observer's (β, κ) and again obtains

$$Q^2 = \beta^2 + \kappa^2.$$

Thus Q is the *norm* of Total Relational Shift, not a spatial distance. Geometrically, the observer is always at the centre of its own S^1 (or S^2) carrier, and any external system is a point (β, κ) on that plane. The scalar Q measures the total deviation from the observer's relational origin.

Remark 6.1 (Closure-specific simplification). *Under energetic closure $\kappa^2 = 2\beta^2$ (circular/periodic systems), the norm reduces to $Q^2 = 3\beta^2$ (proved uniquely in Chapter, 9.3).*

In general (open or elliptic) configurations, the full definition $Q^2 = \beta^2 + \kappa^2$ must be used.

6.1 Principle of Relational Reciprocity

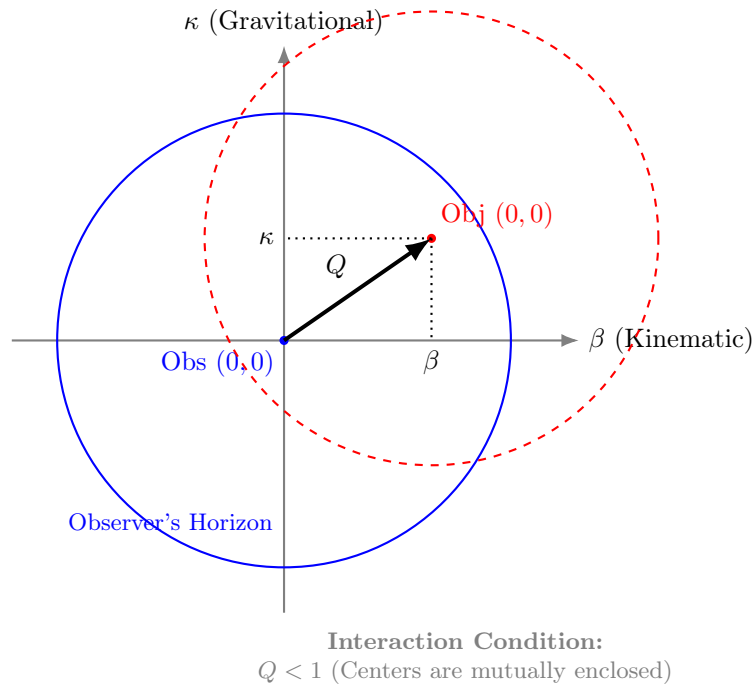


Figure 1: **Relational Self-Centering.** The total shift Q is defined by the orthogonal projections β and κ . Interaction is causal only when the center of the Object lies within the Observer's horizon ($Q < 1$), ensuring mutual coverage.

Self-centering reciprocity. Every observer performs self-centering:

$$(\beta, \kappa) = (0, 0).$$

When I observe another system, I assign to it (β, κ) and therefore a total shift

$$Q^2 = \beta^2 + \kappa^2.$$

When that system observes me, it again self-centres and obtains the same form. It assigns to me some (β, κ) and again computes $Q^2 = \beta^2 + \kappa^2$.

Reciprocity is therefore not a vector symmetry in a shared space. It is a symmetry of the *self-centering operation*: each observer applies the same rule and only the norm of shift is invariant.

Summary

Relational reciprocity = invariance of the norm Q under self-centering.

There is no common background arena. There are only mutual total shift magnitudes Q computed in each observer's own relational origin.

7 Clear Relational Symmetry Between Kinematic and Potential Projections

On the unit kinematic circle (S^1) we parameterize

$$(\beta, \beta_Y) = (\cos \theta_1, \sin \theta_1),$$

so that the invariant internal phase projection reads

$$E \beta_Y = E_0 \implies \boxed{E = \frac{E_0}{\beta_Y} = \frac{E_0}{\sin \theta_1}}, \quad p = \frac{E}{c} \beta = \frac{E_0 \beta}{\beta_Y} = E_0 \cot \theta_1,$$

and therefore $E^2 = (pc)^2 + E_0^2$.

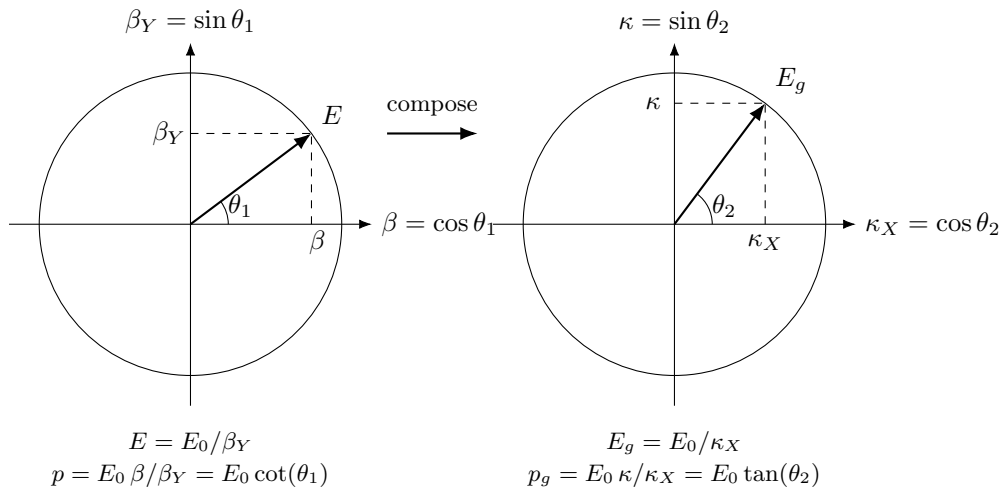
On the gravitational circle (S^2) we parameterize

$$(\kappa_X, \kappa) = (\cos \theta_2, \sin \theta_2),$$

so that the invariant internal phase projection reads

$$E_g \kappa_X = E_0 \implies \boxed{E_g = \frac{E_0}{\kappa_X} = \frac{E_0}{\cos \theta_2}}, \quad p_g = E_g \kappa = \frac{E_0 \kappa}{\kappa_X} = E_0 \tan \theta_2,$$

and therefore $E_g^2 = p_g^2 + E_0^2$.



Now we can clearly see the underlying symmetry between relativistic and gravitational factors that can be expressed in unified algebraic and trigonometric forms, as shown in Table 1.

Summary

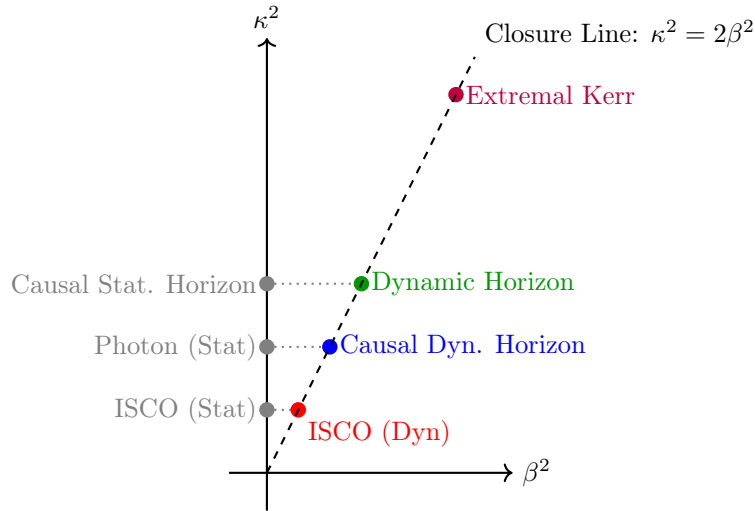
The familiar SR and GR factors emerge here as projections of the same conserved resource. Relativistic (β) and gravitational (κ) modes are not separate "effects" but dual aspects of one energy-transformation constraint revealing their unified origin.

$\theta_1 = \arccos(\beta), \quad \theta_2 = \arcsin(\kappa), \quad \kappa^2 = 2\beta^2$	
Algebraic Form	Trigonometric Form
$\beta = v/c$	$\beta = \cos(\theta_1)$
$\kappa = \sqrt{R_s/r}$	$\kappa = \sin(\theta_2)$
$\beta_Y = \sqrt{1 - \beta^2}$	$\beta_Y = \sin(\theta_1)$
$\kappa_X = \sqrt{1 - \kappa^2}$	$\kappa_X = \cos(\theta_2)$
$p = E_0/c \cdot \beta/\beta_Y$	$p = E_0/c \cdot \cot(\theta_1)$
$p_g = E_0/c \cdot \kappa/\kappa_X$	$p_g = E_0/c \cdot \tan(\theta_2)$
$\tau = \beta_Y \kappa_X$	$\tau = \sin(\theta_1) \cos(\theta_2)$
$\tau_Y = \sqrt{1 - \tau^2}$	$\tau_Y = \sqrt{1 - \tau^2}$
$Q = \sqrt{\kappa^2 + \beta^2} = \sqrt{3}\beta$	$Q = \sqrt{3} \cos(\theta_1)$
$Q_Y = \sqrt{1 - Q^2} = \sqrt{1 - \kappa^2 - \beta^2} = \sqrt{1 - 3\beta^2}$	$Q_Y = \sqrt{1 - 3 \cos^2(\theta_1)}$

Table 2: Unified representation of relativistic and gravitational effects for closed systems.

Phenomenon	Radius r	β^2	κ^2	Q^2	Comment
ISCO (Static location)	$3R_s$	0	$\frac{1}{3}$	$\frac{1}{3}$	Pure coordinate location, $r = R_s/\kappa^2$
ISCO (Dynamic state)	$3R_s$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	Marginal orbital stability, $Q^2 = Q_Y^2 = 1/2$
Photon sphere (Static)	$\frac{3}{2}R_s$	0	$\frac{2}{3}$	$\frac{2}{3}$	Pure coordinate location
Causal Dynamic horizon	$\frac{3}{2}R_s$	$\frac{1}{3}$	$\frac{2}{3}$	1	Relational horizon under closure $\kappa^2 = 2\beta^2$, $Q = 1$
Causal Static horizon	R_s	0	1	1	Purely gravitational closure limit, $\kappa^2 = 1$
Dynamic horizon	R_s	$\frac{1}{2}$	1	$\frac{3}{2}$	Orbital closure at Schwarzschild radius
Extremal Kerr horizon	$\frac{1}{2}R_s$	1	2	3	Maximal rotation, $\beta = 1$, merged horizons

Table 3: Critical radii and their projectional parameters. Static coordinate locations ($\beta = 0$) are strictly distinguished from their corresponding dynamic orbital states that emerge from the closure law $\kappa^2 = 2\beta^2$.



8 Equivalence Principle as Derived Identity

Lemma 8.1 (Unified Relational Scaling). *Within the relational framework of WILL, both kinematic (S^1) and gravitational (S^2) transformations act as independent projections of the same invariant energy E_0 . Each projection rescales the observable quantities by its respective geometric factor:*

$$E = \frac{E_0}{\beta_Y}, \quad E_g = \frac{E_0}{\kappa_X}.$$

Proof. On the kinematic circle S^1 , the invariant vertical projection corresponds to $\beta_Y = \sin \theta_1$. Preserving the same invariant leg E_0 forces the stretch $E/E_0 = 1/\beta_Y$. On the gravitational sphere S^2 , the invariant horizontal projection

is $\kappa_X = \cos\theta_2$, forcing $E_g/E_0 = 1/\kappa_X$. These transformations are independent and commute, each preserving the closure identity of its respective carrier. \square

Theorem 8.2 (Equivalence of Inertial and Gravitational Response). *Composing the independent relational stretches of Lemma 8.1 yields the total local energy scale*

$$E_{\text{loc}} = \frac{E_0}{\tau} = \frac{E_0}{\beta_Y \kappa_X} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}}.$$

The corresponding inertial and gravitational projections share a single operational factor,

$$\tilde{p} = \frac{E_{\text{loc}}}{c} \beta, \quad \tilde{p}_g = \frac{E_{\text{loc}}}{c} \kappa,$$

both governed by the same effective mass

$$m_{\text{eff}} = \frac{E_0}{\beta_Y \kappa_X c^2} = \frac{E_0}{\tau c^2}.$$

Therefore,

$$m_g \equiv m_i \equiv m_{\text{eff}},$$

and the Einstein equivalence principle follows as a necessary structural identity of WILL.

Corollary 8.3 (Mass Invariance under Relational Scaling). *The invariant core E_0 denotes the complete internal equilibrium state ($\beta_Y = \kappa_X = 1$). Relational factors β_Y and κ_X rescale only external manifestations (energy, momentum, and rates), while E_0 remains unchanged. Hence,*

$$m_g \equiv m_i \equiv m = E_0/c^2,$$

is not a dynamical statement but the definition of rest invariance itself.

Remark 8.4 (Composition-Independence). *Decomposing the invariant rest energy into internal channels,*

$$E_0 = \sum_a E_0^{(a)},$$

each term couples identically through the same geometric stretch:

$$E_{\text{loc}} = \sum_a \frac{E_0^{(a)}}{\tau}.$$

Since all channels scale by the same factor $1/\tau = 1/(\beta_Y \kappa_X)$, ratios between channels cancel in all observables. Therefore, composition-independence of motion (Eotvos universality) follows identically, without requiring a postulate $m_g = m_i$.

Remark 8.5 (Quantum Interface). *The relational phase increment inherits the same scaling:*

$$\Delta\phi \propto E_{\text{loc}} \Delta\lambda,$$

where $\Delta\lambda$ is the internal ordering parameter. Thus both kinematic and gravitational phase shifts share the same stretch $1/\tau = 1/(\beta_Y \kappa_X)$, yielding composition-independent matter-wave interference patterns.

Summary:

In WILL, the equivalence of inertial and gravitational mass is not assumed but follows necessarily from the compositional closure of relational geometry. What General Relativity posits as a postulate, WILL reveals as a corollary.

9 Closure: Relational Origin of the Virial Theorem

Having established that directional (kinematic) and omnidirectional (potential) relations are carried by the S^1 and S^2 respectively, we now derive the relationship that unifies them.

9.1 Derivation of the Energetic Closure Condition

Remark 9.1 (From slice to whole S^2). *Although we parametrise a single meridional great circle (κ_X, κ) for algebraic convenience, the amplitude κ^2 denotes the total omnidirectional budget of the S^2 carrier. The exchange-rate factor 2 reflects that S^2 has two independent relational degrees of freedom even when calculations are carried out on a representative great-circle section.*

9.2 Uniqueness of the Exchange Rate (No Hidden Weighting)

Lemma 9.2 (DOF-Indifference). *Under maximal symmetry (no privileged directions) and ontological minimalism (no hidden structure), any admissible conserved budget must assign equal quadratic weight to each independent relational degree of freedom.*

Proof. If two independent DOF contributed unequal weights to the conserved quadratic budget, then the theory would contain an implicit weighting structure that distinguishes DOF. This constitutes a privileged feature not derivable from relations, violating maximal symmetry and minimalism. \square

Theorem 9.3 (Closure). *Within WILL, the only exchange rate between the kinematic carrier S^1 (1 DOF) and the gravitational carrier S^2 (2 DOF) compatible with (i) closure in quadratic form, (ii) maximal symmetry, and (iii) ontological minimalism is:*

$$\boxed{\kappa^2 = 2\beta^2.}$$

Proof. Let b denote the conserved quadratic budget associated with a single independent DOF. By Lemma 9.2, each DOF must contribute the same amount b .

The carrier S^1 has 1 DOF, so its total quadratic budget is $B_{S^1} = 1 \cdot b = b$. The carrier S^2 has 2 DOF, so its total quadratic budget is $B_{S^2} = 2 \cdot b$.

An exchange rate is precisely the statement that the omnidirectional budget is a fixed multiple of the directional budget:

$$B_{S^2} = \mathcal{R} B_{S^1}.$$

Substituting $B_{S^2} = 2b$ and $B_{S^1} = b$ gives $\mathcal{R} = 2$, hence

$$\kappa^2 = \mathcal{R} \beta^2 = 2\beta^2.$$

Any $\mathcal{R} \neq 2$ would require unequal DOF weighting (hidden structure) or an extra free parameter, violating the methodology. \square

Remark 9.4 (Status). *The factor 2 is not empirical calibration and not a coordinate choice. It is the unique consequence of (1 DOF vs 2 DOF) under symmetry and minimalism.*

Definition 9.5 (Closure Factor).

$$\delta \equiv \frac{\kappa^2}{2\beta^2}$$

A subsystem is energetically closed if $\langle \delta \rangle_{\text{cycle}} = 1$. For circular orbits, $\delta \equiv 1$.

Corollary 9.6 (Energetic Closure Criterion). *Closed systems (momentary or periodic) satisfy $\kappa^2 = 2\beta^2$ identically. Open systems display $\delta \neq 1$, the magnitude of which quantifies the energy flow through unaccounted channels. When all channels are included, closure is restored.*

Remark 9.7 (Physical Interpretation). *The exchange rate between the kinematic and gravitational projections corresponds to the ratio of their relational dimensions. This purely geometric constant (2) replaces the empirical proportionalities of classical dynamics. It is the relational origin of the **virial theorem**: the kinetic and potential aspects of WILL maintain closure through the invariant ratio*

$$\boxed{\kappa^2 = 2\beta^2.}$$

Illustrative Examples.

- **Circular Orbit (Closed).** A body at any orbital phase satisfies $\kappa^2 = 2\beta^2$. The entire conserved resource is partitioned between kinetic and gravitational projections; no internal "breathing" and no external channel exists.
- **Elliptical Orbit (Closed).** A body satisfies $\langle \kappa^2 \rangle = 2 \langle \beta^2 \rangle$ as an average per orbital cycle due to internal "breathing" of elliptical systems. Though this internal "breathing" is restricted by the Energy-Symmetry Law (10) so the difference $W = \frac{1}{2}(\kappa^2 - \beta^2) = \text{constant}$ at any orbital phase (??). No external channel exists.
- **Radiating Binary (Open).** An elliptical compact binary violates $\langle \kappa^2 \rangle = 2 \langle \beta^2 \rangle$ when only orbital degrees of freedom are counted, the closure defect δ quantifying energy lost to gravitational radiation. Including all channels restores closure.

Summary:

1. WILL is a closed relational structure, SPACETIME \equiv ENERGY.
2. The simplest maximally symmetric carriers of these relations are S^1 and S^2 .
3. The parameters $\beta = \cos \theta_1$ and $\kappa = \sin \theta_2$ are thus constrained to these carriers.
4. The geometric exchange rate between these modes equals the ratio of their DOF: 2.

Remark 9.8 (Geometric Origin of Physical Law). *The relation between kinetic and potential energy is not an empirical coincidence but a geometric necessity of relational closure. Classical mechanics merely approximates this deeper invariant. Explicitly,*

$$\boxed{\text{Geometric Distribution } (\kappa^2) \equiv 2 \times \text{Kinetic Distribution } (\beta^2).}$$

10 Energy-Symmetry Law

In RG, every transformation is bidirectional: each change observed by A corresponds to an equal and opposite change observed by B . This reciprocity is the algebraic form of causal continuity, and its geometric expression is the Energy-Symmetry Law.

10.1 Causal Continuity and Energy Symmetry

Theorem 10.1 (Energy Symmetry). *The specific energy differences (per unit of rest energy) perceived by two observers for a transition between their states balance according to the Energy-Symmetry Law:*

$$\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0. \quad (12)$$

Proof. Consider two observers:

- Observer A at rest on the surface at radius r_A (state defined by $\kappa_A, \beta_A = 0$).
- Observer B orbiting at radius $r_B > r_A$ with orbital velocity v_B (state defined by κ_B, β_B).

Each observer perceives energy transfers as the sum of the change in potential and kinetic energy budgets.

From A 's perspective (transition from surface to orbit):

1. An object gains potential energy by moving away from the gravitational center.
2. It gains kinetic energy by accelerating to orbital velocity.

The total specific energy required for this transition is the sum of these two contributions:

$$\boxed{\Delta E_{A \rightarrow B} = \underbrace{\frac{1}{2}(\kappa_A^2 - \kappa_B^2)}_{\text{Change in Potential}} + \underbrace{\frac{1}{2}(\beta_B^2 - \beta_A^2)}_{\text{Change in Kinetic}}} \quad (13)$$

Since observer A is at rest, $\beta_A = 0$, and the expression simplifies to:

$$\Delta E_{A \rightarrow B} = \frac{1}{2}((\kappa_A^2 - \kappa_B^2) + \beta_B^2) \quad (14)$$

From B 's perspective (transition from orbit to surface):

1. The object loses potential energy descending into a stronger gravitational field.
2. It loses kinetic energy by reducing its velocity to rest.

This results in a specific energy difference:

$$\Delta E_{B \rightarrow A} = \frac{1}{2}((\kappa_B^2 - \kappa_A^2) + (\beta_A^2 - \beta_B^2)) = \frac{1}{2}((\kappa_B^2 - \kappa_A^2) - \beta_B^2) \quad (15)$$

Summing these transfers gives:

$$\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0 \quad (16)$$

Thus, no net energy is created or destroyed in a closed cycle of transitions, confirming the Energy-Symmetry Law as a direct consequence of the closed geometry. \square

10.2 The Specific Energy Transfer (ΔE):

This is the projectional energy difference between states, equivalent to the Total Relational Shift Q , corresponding to the classical total energy of a transition (per unit rest energy). It is defined as the **sum of the changes** in the potential and kinetic energy budgets:

$$\Delta E_{A \rightarrow B} = \Delta U_{A \rightarrow B} + \Delta K_{A \rightarrow B} = \frac{1}{2}(\kappa_A^2 - \kappa_B^2) + \frac{1}{2}(\beta_B^2 - \beta_A^2) \quad (17)$$

It is this quantity, ΔE , that is conserved and must balance to zero in any closed system.

When the closure condition for stable, periodic orbits ($\kappa^2 - 2\beta^2 = 0$) is applied, the general Energy-Symmetry Law simplifies into remarkably elegant and direct forms. These simplified equations provide the precise energy balance for transitions involving energetically closed systems, such as planets or satellites in stable orbits.

Case 1: Surface-to-Orbit Transfer. For a transfer from a state of rest (A, where $\beta_A = 0$) to a closed orbit (B) where E_{0B} is the objects rest energy, the specific energy balance is given by:

$$\frac{E_{A \rightarrow B}}{E_{0B}} = \frac{1}{2}(\kappa_A^2 - \beta_B^2) \quad (18)$$

This result is derived by applying the closure condition $\kappa_B^2 = 2\beta_B^2$ to the general energy transfer formula, elegantly linking the initial potential projection to the final kinetic projection.

Case 2: Orbit-to-Orbit Transfer. For a transfer between two different closed orbits (A and B), the simplification is even more profound. The specific energy balance reduces to:

$$\frac{E_{A \rightarrow B}}{E_{0B}} = \frac{1}{2}(\beta_A^2 - \beta_B^2) \quad (19)$$

In this case, applying the closure condition to both the initial and final orbits causes the potential projection terms (κ^2) to cancel out completely. The entire energy balance of the transfer is expressed purely as the difference between the squares of the initial and final kinetic projections. This demonstrates a deep symmetry in the energetic structure of stable orbital systems.

10.3 Physical Meaning of the Factor $\frac{1}{2}$

The factor $\frac{1}{2}$ originate from the quadratic nature of the energy budgets in RG. The energetic significance of a state is proportional to the **square** of its geometric projection. This is the unavoidable consequence of relational carriers closure condition (amplitude² + phase² = 1). By using only amplitudes (β^2 and κ^2) we operating with half's relational budgets of S^1 and S^2 carriers.

The individual energy budgets are:

- **Specific Potential Energy Budget:** $U/E_0 \propto -\frac{1}{2}\kappa^2$
- **Specific Kinetic Energy Budget:** $K/E_0 = \frac{1}{2}\beta^2$

The factor $\frac{1}{2}$ arises naturally when representing a conserved quantity (energy) through a quadratic measure (the square of a projection). The Energy-Symmetry Law deals with the sum of the *changes* in these individual budgets.

10.4 Universal Speed Limit as a Consequence of Energy Symmetry

Theorem 10.2 (Universal Speed Limit). *The universal speed limit ($v \leq c$) emerges naturally from the requirement of energetic symmetry.*

Proof. Assume an object could exceed the speed of light, implying $\beta > 1$. In this scenario, its specific kinetic energy budget, $\frac{1}{2}\beta^2$, would become arbitrarily large.

The energy transfer required to reach this state, $\Delta E_{A \rightarrow B}$, would also become arbitrarily large. Consequently, no finite physical process could provide a balancing reverse transfer, $\Delta E_{B \rightarrow A}$, that would sum to zero. The fundamental symmetry would be broken:

$$\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} \neq 0 \quad (20)$$

Therefore, the condition $\beta \leq 1$ (which implies $v \leq c$) is an intrinsic requirement for maintaining the causal and energetic consistency of the relational universe. \square

10.5 Single-Axis Energy Transfer and the Nature of Light

Theorem 10.3 (Single-Axis Transformation). *For light, the kinematic projection amplitude reaches its full extent:*

$$\boxed{\beta = 1 \implies \beta_Y = 0.}$$

*This means that all transformation of the relational resource occurs along a **single X axis** $\implies \beta = 1$. The orthogonal Y axis is absent $\implies \beta_Y = 0$, and the total resource of transformation is entirely expressed on one geometric component.*

Proof. For massive bodies, the Energy-Symmetry Law (Section 10) partitions the transformation resource equally between orthogonal axes. The specific binding energy invariant for orbital motion is therefore:

$$W_{\text{mass}} = \frac{1}{2}(\kappa^2 - \beta^2), \quad (21)$$

where the factor $\frac{1}{2}$ arises from this dual-axis distribution (Theorem 3.7). This invariant is conserved for closed systems and reduces to the classical Keplerian energy under the closure condition $\kappa^2 = 2\beta^2$ (Corollary 9.6).

Now consider light. By the Single-Axis Transformation Principle (Theorem 10.3), its kinematic projection saturates the carrier:

$$\beta = 1 \implies \beta_Y = 0.$$

Geometrically, this collapses the relational architecture:

- The Y-axis (Phase component) vanishes entirely, as $\beta_Y = 0$ indicates no internal state evolution.
- No rest frame exists for self-centering (Section 6), eliminating the dual-framing that justifies the $\frac{1}{2}$ partitioning.
- The entire transformation resource concentrates on the single X-axis (Amplitude component).

Consequently, the energy invariant for photon interactions with a massive body (projection κ) must exclude the partitioning factor:

$$W_\gamma = \kappa^2 - \beta^2 = \kappa^2 - 1. \quad (22)$$

This is verified with no gravity state ($\kappa = 0$):

$$\begin{aligned} W_{\text{mass}} &= \frac{1}{2}(0^2 - 0^2) = 0, \\ W_\gamma &= 0^2 - 1^2 = -1. \end{aligned}$$

The value $W_\gamma = -1$ corresponds to the full rest energy cost of transitioning to a photon state - whereas the $\frac{1}{2}$ factor would erroneously yield $-\frac{1}{2}$, violating energy symmetry.

For a concrete test case, consider a photon interacting with a massive body where $\kappa_M = 0.4$:

$$\begin{aligned} W_{\text{mass}} &= \frac{1}{2}(0.4^2 - 1^2) = \frac{1}{2}(-0.84) = -0.42, \\ W_\gamma &= 0.4^2 - 1^2 = -0.84 = 2 \times W_{\text{mass}}. \end{aligned}$$

Thus, the gravitational effect on light is twice that on massive particles - matching the observed factor in light deflection.

The general energy potential follows directly: for light, the full geometric resource expresses unpartitioned along the single axis:

$$\Phi_\gamma = \kappa^2 c^2,$$

while massive bodies retain the partitioned form:

$$\Phi_{\text{mass}} = \frac{1}{2}\kappa^2 c^2.$$

This factor-of-2 is the geometric signature of axis count in relational space. No auxiliary approximations or background structures are introduced; the result emerges from the topological constraint $\beta_Y = 0$ applied to the closed carrier S^1 . □

Summary

Energy Symmetry Law $\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0$ - universal relational energy bookkeeping.

The Speed of Light is the boundary beyond which the energy symmetry law breaks down.

Causality is built-in feature of Relational Geometry.

Light has no rest frame. The disappearance of the Phase component (Y-axis $\beta_Y = 0$) concentrates the entire transformation resource on a single geometric component. This eliminates the $\frac{1}{2}$ partitioning factor, yielding $\Phi_\gamma = \kappa^2 c^2$ and explaining why light experiences twice the geometric effect of massive bodies.

This explains the experimentally verified factor of 2 in gravitational lensing and Shapiro delay, traditionally requiring full General Relativity to derive.

11 Classical Mechanics

A striking consequence of the Energy–Symmetry Law (Section 10) emerges when analysing the total specific orbital energy. Since energy in RG is defined *relationally, as the measure of difference between two states*, we naturally select these two states (e.g., the surface of the central body 'A' and the orbit 'B') as the reference points for the potential and kinetic energy budgets. Under this relational approach, the total specific orbital energy (potential + kinetic, per unit rest mass) naturally appears in a form **structurally identical to the Minkowski interval**.

11.1 Classical Result with Surface Reference

For a test body of mass m on a circular orbit of radius a about a central mass M_\oplus (Earth in our example), classical Newtonian mechanics gives:

$$\Delta U = -\frac{GM_\oplus m}{a} + \frac{GM_\oplus m}{R_\oplus}, \quad (23)$$

$$K = \frac{1}{2}m \frac{GM_\oplus}{a}. \quad (24)$$

Adding these and dividing by the rest–energy $E_0 = mc^2$ yields the dimensionless total:

$$\frac{E_{\text{tot}}}{E_0} = \frac{GM_\oplus}{R_\oplus c^2} - \frac{1}{2} \frac{GM_\oplus}{ac^2}. \quad (25)$$

11.2 Projection Parameters and Minkowski-like Form

Remark 11.1. Here we explicitly define our projections using classical language to ease understanding for readers not yet familiar with WILL RG. For more explicit comparison and clearer derivations, we adopt standard Newtonian notation and define the projection parameters κ^2 and β^2 through their legacy equivalents involving G , M , and c . This is purely a pedagogical choice—the projections themselves remain dimensionless geometric quantities independent of these conventional constants.

Define the WILL projection parameters for the surface and the orbit:

$$\kappa_\oplus^2 \equiv \frac{2GM_\oplus}{R_\oplus c^2}, \quad (26)$$

$$\beta_{\text{orbit}}^2 \equiv \frac{GM_\oplus}{ac^2}. \quad (27)$$

Substituting into (11.1) gives the exact identity:

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}(\kappa_\oplus^2 - \beta_{\text{orbit}}^2). \quad (28)$$

This is already in the form of a *hyperbolic difference of squares*: if we set $x \equiv \kappa_\oplus$ and $y \equiv \beta_{\text{orbit}}$, then

$$\frac{E_{\text{tot}}}{E_0} = \frac{1}{2}(x^2 - y^2), \quad (29)$$

which is structurally identical to a Minkowski interval in $(1 + 1)$ dimensions, up to the constant factor $\frac{1}{2}$.

Sign convention. We use $U/E_0 = -\frac{1}{2}\kappa^2$ and $K/E_0 = \frac{1}{2}\beta^2$ as *budgets*. The minus sign attaches to the potential budget by convention of reference (surface vs infinity); the budgets themselves are positive quadratic measures, while transfer ΔE is the signed sum of budget *changes*.

11.3 Physical Interpretation

In classical derivations, (11.1) is just the sum $\Delta U + K$ with a particular choice of potential zero. In the RG, (11.2) emerges directly from the energy–symmetry relation:

$$\Delta E_{A \rightarrow B} = \frac{1}{2}((\kappa_A^2 - \kappa_B^2) + \beta_B^2),$$

with $(A, B) = (\text{surface}, \text{orbit})$, and is *invariantly* expressible as a difference of squared projections.

This shows that the Keplerian total energy is not an isolated Newtonian artifact but a special case of a deeper geometric structure. While RG refuse to postulate any spacetime metric in the traditional sense, the emergence of this Minkowski-like structure from purely energetic principles is a powerful indicator of the deep identity between the geometry of spacetime and the geometry of energy transformation.

Why This Matters

- In classical form, the total orbital energy per unit mass depends only on GM and a , and is independent of the test-mass m .
- In WILL form, the same fact is embedded in Energy–Symmetry difference of squared projections, with no need for separate “gravitational” and “kinetic” constructs.
- This re framing answers *why* the Keplerian combination appears: it is enforced by the underlying geometry of energy transformation.

12 Lagrangian Hamiltonian and Newton’s Third Law

The following section present philosophical and algebraic demonstration: the standard L and H arise as degenerate limits of the relational Energy-Symmetry law.

We now demonstrate that the familiar Lagrangian and Hamiltonian formalisms arise as limiting cases of the two-point relational Energy-Symmetry Law. Specifically, they emerge when the relational structure between two distinct observers A and B is collapsed into a single-point local description. This collapse preserves computational utility but reduces the ontological transparency of the underlying relational structure.

12.1 Definitions of Parameters

We consider a central mass M and a test mass m . The state of the test mass is described in polar coordinates (r, ϕ) relative to the central mass.

- r_A — reference radius associated with observer A (e.g., planetary surface).
- r_B — orbital radius of the test mass m (position of observer B).
- $v_B^2 = \dot{r}_B^2 + r_B^2 \dot{\phi}^2$ — total squared orbital speed at B .
- $\beta_B^2 = v_B^2/c^2$ — dimensionless kinematic projection at B .
- $\kappa_A^2 = 2GM/(r_A c^2)$ — dimensionless potential projection defined at A .

12.2 The Relational Lagrangian

Instead of a relational energy, we define the *clean relational Lagrangian* L_{rel} , which represents the kinetic budget at point B relative to the potential budget at point A :

$$L_{\text{rel}} = T(B) + U(A) = \frac{1}{2}m(\dot{r}_B^2 + r_B^2 \dot{\phi}^2) + \frac{GMm}{r_A}. \quad (30)$$

In dimensionless form, using the rest energy $E_0 = mc^2$, this is:

$$\frac{L_{\text{rel}}}{E_0} = \frac{1}{2}(\beta_B^2 + \kappa_A^2). \quad (31)$$

This two-point, relational form is the clean geometric statement.

12.3 First Ontological Collapse: The Newtonian Lagrangian

If one commits the first ontological violation by identifying the two distinct points, $r_A = r_B = r$, the relational structure degenerates into a local, single-point function:

$$L(r, \dot{r}, \dot{\phi}) = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \quad (32)$$

This is precisely the standard Newtonian Lagrangian. Its origin is not fundamental but arises from the collapse of the two-point relational Energy Symmetry law into a one-point formalism.

12.4 Second Ontological Collapse: The Hamiltonian

Introducing canonical momenta,

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}, \quad (33)$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}, \quad (34)$$

one defines the Hamiltonian via the Legendre transformation $H = p_r\dot{r} + p_\phi\dot{\phi} - L$. This evaluates to the total energy of the collapsed system:

$$H = T + U = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{GMm}{r}. \quad (35)$$

12.4.1 Interpretation

In terms of the collapsed WILL projections ($\beta^2 = v^2/c^2$ and $\kappa^2 = 2GM/(rc^2)$, both strictly positive), the match to standard mechanics becomes explicit:

$$L = \frac{1}{2}mv^2 + \frac{GMm}{r} \longleftrightarrow \frac{1}{2}mc^2(\beta^2 + \kappa^2), \quad (36)$$

$$H = \frac{1}{2}mv^2 - \frac{GMm}{r} \longleftrightarrow \frac{1}{2}mc^2(\beta^2 - \kappa^2). \quad (37)$$

Here the “+” or “−” signs do not come from κ^2 itself, which is always positive, but from the ontological collapse of the two-point relational energy law into a single-point formalism. In WILL, both projections are clean and positive; in standard mechanics, the apparent sign difference arises only after this collapse.

Both the Lagrangian and Hamiltonian thus emerge from the same relational Energy-Symmetry Law under the identification $r_A = r_B = r$. The apparent sign difference between L and H is not a fundamental feature but an artifact of this single-point collapse: in the full two-point relational law, both projections β^2 and κ^2 are strictly positive.

Remark 12.1 (Mathematical Status: Groupoid vs. Group). *From a category-theoretic perspective, the relational transitions $\Delta E_{A \rightarrow B}$ form a **Groupoid** structure, where operations are defined only between specific connected states. Standard field theories (Hamiltonian/Lagrangian) rely on the collapse of this structure into a **Group** acting on a global manifold (Quotienting). Thus, the relational Energy-Symmetry Law operates at the Groupoid level, prior to the reduction into global field theories.*

Key Message

The Lagrangian and Hamiltonian arise as single-point limiting cases of the two-point relational Energy-Symmetry Law $\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0$. They remain computationally valid within their domain of applicability. RG provides the more general two-point relational structure from which these formalisms can be systematically derived.

Legacy Dictionary (for conventional formalisms).

Within RG, all physical content is expressed purely in terms of relational projections β and κ on S^1 and S^2 . For readers accustomed to standard frameworks, the following translation rules may help:

1. *General Relativity (metric form):*

$$\kappa_X \hat{=} \sqrt{-g_{tt}} \quad (\text{static spacetimes}), \quad \beta \hat{=} \frac{\|u_{\text{spatial}}^\mu\|}{u^t c}.$$

2. *Canonical mechanics (Lagrangian/Hamiltonian)*: Quantities such as $p_i = \partial L / \partial \dot{q}^i$ do not belong to the ontology of RG. They arise only after collapsing the two-point relational law into a one-point formalism. They are secondary derived quantities arising after the two-point relational structure is reduced to a local single-point formalism.

Here the symbol $\hat{=}$ denotes not an ontological identity, but a pragmatic dictionary entry for translation into legacy notation.

12.5 Third Ontological Collapse: Newton's Third Law

We now demonstrate that Newton's Third Law, like the Lagrangian and Hamiltonian, emerges as a limiting case of RG. It arises as a necessary mathematical consequence of the same ontological collapse — forcing a two-point relational law into a single-point, instantaneous formalism.

Theorem 12.2 (Newton's Third Law as a Single-Point Limit). *The Energy-Symmetry Law ($\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0$) mathematically necessitates Newton's Third Law ($\vec{F}_{AB} = -\vec{F}_{BA}$) in the classical, non-relativistic limit where the two-point relational energy budget is collapsed into a single-point potential function $U(\vec{r})$.*

Proof. We begin with the foundational Energy-Symmetry Law (Section 10), the principle of causal balance for state transitions:

$$\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0.$$

In the classical, non-relativistic limit, this two-point relational law is "ontologically collapsed" into a single-point potential energy function, U . This function is assumed to depend only on the relative positions of the two interacting entities, A and B :

$$U = U(\vec{r}) \quad \text{where} \quad \vec{r} = \vec{r}_B - \vec{r}_A.$$

This $U(\vec{r})$ is the classical approximation of the system's relational energy budget. In this collapsed formalism, the force \vec{F} is defined as the negative gradient of this potential.

(1) **Force on B by A (\vec{F}_{AB}):** This force is found by taking the gradient with respect to B 's coordinates:

$$\vec{F}_{AB} = -\nabla_B U(\vec{r}_B - \vec{r}_A) \tag{38}$$

$$= -\left(\frac{dU}{d\vec{r}}\right) \cdot \left(\frac{\partial \vec{r}}{\partial \vec{r}_B}\right) \tag{39}$$

$$= -\nabla U(\vec{r}) \cdot (\mathbf{I}) \tag{40}$$

$$= -\nabla U(\vec{r}) \tag{41}$$

(2) **Force on A by B (\vec{F}_{BA}):** This force is found by taking the gradient with respect to A 's coordinates:

$$\vec{F}_{BA} = -\nabla_A U(\vec{r}_B - \vec{r}_A) \tag{42}$$

$$= -\left(\frac{dU}{d\vec{r}}\right) \cdot \left(\frac{\partial \vec{r}}{\partial \vec{r}_A}\right) \tag{43}$$

$$= -\nabla U(\vec{r}) \cdot (-\mathbf{I}) \tag{44}$$

$$= +\nabla U(\vec{r}) \tag{45}$$

(3) **Conclusion:** By direct comparison of the results, we find:

$$\vec{F}_{AB} = -\nabla U(\vec{r}) \quad \text{and} \quad \vec{F}_{BA} = +\nabla U(\vec{r}).$$

Therefore, within the single-point collapsed formalism:

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}}$$

This completes the proof. Since all physical observations are strictly relative, embedding the energy budget within an absolute spatial container (\vec{r}_A, \vec{r}_B) introduces unobservable parameters, which strictly violates the principle of Epistemic Hygiene. To remain empirically grounded, the collapsed potential U must depend exclusively on the measurable physical relation $\vec{r}_B - \vec{r}_A$. Consequently, the principle of equal and opposite forces emerges not as an empirical axiom of mechanics, but as the mathematical signature of any causally balanced, background-independent system $\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0$. \square

Philosophical Consequences

[Substantialism vs. Relationalism](#)

13 Relational Orbital Mechanics

short summary with the link to ROM doc

14 Density, Mass, and Pressure

14.1 Derivation of Density

Translating RG (2D) to Conventional Density (3D). In RG κ^2 is the 2D parameter defined in the relational carrier S^2 . In conventional physics, the source term is volumetric density ρ , a 3D concept defined by the "cultural artifact" (a Newtonian "cannonball" model) of mass-per-volume.

To bridge our 2D theory with 3D empirical data, we must create a "translation interface". We do this by explicitly adopting the conventional (Newtonian) definition of density, $\rho \propto M/r^3$, as our "translation target".

From the projective analysis established in the previous sections:

$$\kappa^2 = \frac{R_s}{r},$$

where κ emerges from the energy projection on the area of unit sphere S^2 , and $R_s = 2GM/c^2$ links to the mass scale factor $M = E_0/c^2$.

This leads to mass definition:

$$M = \frac{\kappa^2 c^2 r}{2G}$$

To translate this into a volumetric density, we first adopt the conventional 3D (volumetric) proxy, r^3 . This is not a postulate of RG, but the first step in applying the legacy (3D) definition of density:

$$\frac{M}{r^3} = \frac{\kappa^2 c^2}{2Gr^2}$$

This expression, however, is incomplete. Our κ^2 "lives" on the 2D surface S^2 (which corresponds to 4π), while the r^3 proxy implicitly assumes a 3D volume. To correctly normalize the 2D parameter κ^2 against the 3D volume, we must apply the geometric normalization factor of the S^2 carrier by dividing on to area of the sphere, which is $1/4\pi$.

This normalization is the necessary geometric step to interface the 2D relational carrier (S^2) with the 3D legacy definition of density:

$$\rho = \frac{1}{4\pi} \left(\frac{\kappa^2 c^2}{2Gr^2} \right)$$

$$\rho = \frac{\kappa^2 c^2}{8\pi Gr^2}$$

Local Density \equiv Relational Projection

Maximal Density. At $\kappa^2 = 1$ (the horizon condition (for non rotating systems), $r = R_s$), this density reaches its natural bound, ρ_{\max} , which is derived purely from geometry:

$$\rho_{\max} = \frac{c^2}{8\pi Gr^2}$$

Normalized Relation. Thus, our "translation" reveals an identity: the geometric projection κ^2 is the ratio of density to the maximal density:

$$\boxed{\kappa^2 = \frac{\rho}{\rho_{\max}} \quad \text{arrow} \quad \kappa^2 \equiv \Omega}$$

14.2 Self-Consistency Requirement

The mass scale factor can be expressed in two equivalent ways.

From the geometric definition:

$$M = \frac{\kappa^2 c^2 r}{2G}$$

From the energy density:

$$M = \alpha r^n \rho.$$

Substituting $\rho = \frac{\kappa^2 c^2}{8\pi G r^2}$ into $M = \alpha r^n \rho$ gives

$$M = \frac{\alpha \kappa^2 c^2 r^{n-2}}{8\pi G}.$$

Equating the two forms:

$$\frac{\alpha r^{n-2}}{8\pi} = \frac{r}{2}.$$

For the mass M to remain a constant independent of the measurement scale r , the exponent must be $n = 3$, yielding $\alpha = 4\pi$. Hence,

$$M = 4\pi r^3 \rho,$$

which closes the consistency loop between the geometric and density-based formulations.

Desmos Project

[Derivation of Density and Pressure](#)

14.3 Pressure as Surface Curvature Gradient

In the RG framework pressure is not a thermodynamic assumption but the direct consequence of curvature gradients. The radial balance relation gives

$$P(r) = \frac{c^4}{8\pi G} \frac{1}{r} \frac{d\kappa^2}{dr}.$$

Using $\kappa^2 = R_s/r$, one finds $d\kappa^2/dr = -\kappa^2/r$, hence

$$P(r) = -\frac{\kappa^2 c^4}{8\pi G r^2}.$$

Since the local energy density is

$$\rho(r) = \frac{\kappa^2 c^2}{8\pi G r^2},$$

this yields the invariant equation of state

$$P(r) = -\rho(r) c^2.$$

Interpretation. P is a surface-like negative pressure (isotropic tension), not a bulk volume pressure. It expresses the resistance of energy-geometry itself to changes in projection.

Consistency. If one formally freezes the projection parameter ($d\kappa^2/dr = 0$), then $P = 0$. But in this case the angular curvature terms remain uncompensated, and the field equation is no longer satisfied. Any nontrivial radial dependence of κ inevitably generates the negative tension

$$P = -\rho c^2,$$

which precisely cancels the residual curvature. Thus the negative pressure is not optional but a necessary ingredient for full self-consistency.

Desmos Project

[Derivation of Density and Pressure](#)

Maximum pressure. At the geometric bound $\kappa^2 = 1$ (horizon condition), the density saturates at

$$\rho_{\max} = \frac{c^2}{8\pi G r^2},$$

and the corresponding pressure is

$$P_{\max} = -\rho_{\max} c^2 = -\frac{c^4}{8\pi G r^2}.$$

This negative surface pressure represents the ultimate tension limit of spacetime fabric at a given scale r .

Pressure in WILL is the intrinsic surface tension of energy-geometry, saturating at $P_{\max} = -c^4/(8\pi G r^2)$.

Physical meaning

The negative pressure is not an exotic substance but the geometric tension required to maintain self-consistency when κ^2 varies radially. It's the relational analogue of surface tension in a soap bubble.

15 Unified Geometric Field Equation

From the energy-geometry equivalence, the complete description of gravitational phenomena reduces to a single algebraic relation linking the geometric scale to the energy density ratio:

$$\kappa^2 = \frac{R_s}{r} = \frac{\rho_{field}}{\rho_{max}}$$

This identity defines the **local energy state of the geometry itself**. Here $\rho_{max} = c^2/(8\pi Gr^2)$ is the saturation density limit, and ρ_{field} is the effective energy density of the relational curvature.

15.1 Field Equation and Matter Sources

For a static, spherically symmetric configuration containing matter with density $\rho_{matter}(r)$, the relationship is governed by the differential accumulation of the potential:

$$\frac{d}{dr}(r\kappa^2) = \frac{8\pi G}{c^2} r^2 \rho_{matter}(r) \quad (46)$$

This expression reproduces the tt -component of the Einstein field equations.

The Vacuum Solution ($\rho_{matter} = 0$). In the vacuum region outside a central mass, the source density vanishes ($\rho_{matter} = 0$). The field equation implies conservation of the projection budget:

$$\frac{d}{dr}(r\kappa^2) = 0 \implies r\kappa^2 = \text{const} = R_s.$$

Thus, we recover the potential law of WILL RG:

$$\kappa^2 = \frac{R_s}{r}.$$

Resolution of Roles

1. **The Identity** $\kappa^2 = \rho/\rho_{max}$ describes the state of the *field* geometry.
2. **The Equation** $(r\kappa^2)' \sim \rho_{matter}$ describes how *matter* generates that geometry.
In vacuum, the generator is zero, but the field persists as the algebraic structure $\kappa^2 = R_s/r$.

15.2 No Singularities, No Hidden Regions

This framework introduces no interior singularities, no coordinate patches hidden behind horizons, and no ambiguous initial conditions. The geometric field equation:

$$\frac{R_s}{r} = \frac{8\pi G}{c^2} r^2 \rho = \kappa^2$$

ensures that curvature and energy density evolve smoothly and remain bounded across all observable scales.

WILL Relational Geometry resolves the singularity problem not by regularizing divergent terms, nor by introducing quantum effects, but by geometrically constraining the domain of valid projections. Curvature is always finite, and energy remains bounded by construction. Black holes become energetically saturated but non-singular regions, described entirely by finite, dimensionless parameters.

This projectional approach provides a clean, intrinsic termination to gravitational collapse, replacing singular endpoints with structured, maximally curved boundaries.

- **Surface-scaled closure (vs. volume filling)**. Mass follows the algebraic closure $M = 4\pi r^3 \rho$ with $\rho = \kappa^2 c^2/(8\pi Gr^2)$; the 4π is the spherical projection measure, not a Newtonian volume average.
- **Natural bounds**. The constraint for non rotating systems $\kappa^2 \leq 1$ enforces $\rho \leq \rho_{max}$ and $|P| \leq |P_{max}| = c^4/(8\pi Gr^2)$, avoiding singularities without extra hypotheses.

16 Theoretical Ouroboros

Closure

Ontological principle is proven as the inevitable consequence of geometric consistency. Field Equation \iff Ontological Principle

We have shown that this single Ontological Principle (2.3), through pure geometric reasoning, necessarily leads to an equation which mathematically expresses the very same equivalence we began with. We started with SPACETIME \equiv ENERGY, from which geometry and physical laws are logically derived, and these derived laws then loop back to intrinsically define and limit the very nature of energy and spacetime, proving the self-consistency of the initial idea.

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY.}}$$

The ratio of geometric scales equals the ratio of energy densities.

$$\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}$$

$$\boxed{\begin{array}{c} \text{SPACETIME GEOMETRY} \\ \equiv \\ \text{ENERGY DISTRIBUTION} \end{array}}$$

Summary

All physical structure emerges from the single relational equivalence:

$$\boxed{\text{SPACETIME} \equiv \text{ENERGY}}$$

From this, by enforcing geometric self-consistency, one necessarily arrives at the Unified Geometric Field Equation:

$$\kappa^2 = \frac{R_s}{r} = \frac{\rho}{\rho_{max}}$$

This is not an external law but an intrinsic closure relation: geometry and energy are two mutually defining projections of a single entity. It represents the completion of the theoretical Ouroboros — where the principle generates its own mathematical expression and the expression in turn validates the principle.

From a philosophical and epistemological point of view, this can be considered the crown achievement of any theoretical framework - the "Theoretical Ouroboros". But regardless of the aesthetic beauty of this result, let us remain sceptical.

16.1 WILL: Unity of Relational Structure

The ontological principle

$$\text{SPACETIME} \equiv \text{ENERGY}$$

states that there is only one closed relational resource - WILL. What we call space, time and energy are different projections of the same relational structure. For any energy-closed system observed from a relational origin, this resource appears through four operational projections:

Desmos project

WILL= 1 Derivation of the W_{ILL} invariant.

These quantities are defined as correlated projections of the same underlying WILL structure. In the dimensionful form we write:

$$M = \frac{\beta^2}{\beta_Y} \frac{c^2 a}{G}, \quad E = \frac{\kappa^2}{\kappa_X} \frac{c^4 a}{2G}, \quad T = \kappa_X \left(\frac{2Gm_0}{\kappa^2 c^3} \right)^2, \quad L = \beta_Y \left(\frac{Gm_0}{\beta^2 c^2} \right)^2,$$

where (β, β_Y) and (κ, κ_X) are the kinematic and gravitational projections on the carriers S^1 and S^2 , m_0 is the rest mass, $E_0 = m_0 c^2$ is the rest energy, and a is the relational scale of the system as semi-major axis (average length per cycle within this system).

The closure conditions of the carriers,

$$\beta^2 + \beta_Y^2 = 1, \quad \kappa_X^2 + \kappa^2 = 1,$$

together with the energetic exchange condition

$$\kappa^2 = 2\beta^2,$$

fix a unique dimensionless combination of these four projections. Combining E , T , M , and L we obtain

$$W_{ILL} \equiv \frac{ET}{ML} = \frac{\frac{E_0}{\kappa_X} \kappa_X t^2}{\frac{m_0}{\beta_Y} \beta_Y a^2} = \frac{E_0 t^2}{m_0 a^2}.$$

By the relations that tie temporal and $t = a/c$ and spacial $a = R_s/\kappa^2$ scales this ratio is identically equal to unity for any closed system:

$$W_{ILL} = \frac{ET}{ML} = 1.$$

All dimensionful constants cancel automatically; the value is fixed by the geometry of the carriers, not by a choice of units.

Desmos project

WILL EARTH GPS calculates GPS satellite time shift and testing $W_{ILL} = 1$ on the Earth-GPS system

The same invariant can be written in a phase-normalized form, using local projections

$$E_o = \frac{E_0}{\kappa_{X_o}}, \quad M_o = \frac{m_0}{\beta_{Y_o}}, \quad T_o = \kappa_{X_o} t_o^2, \quad L_o = \beta_{Y_o} r_o^2,$$

Equivalently, for any state (β_o, κ_o) , any scale r_o and any phase along the orbit. Then

$$W_{ILL} = \frac{E_o T_o}{M_o L_o} = 1 \quad \text{for ALL ENERGY'S, ALL SCALES and ALL PHASES.}$$

$$\frac{E_o}{M_o} = \frac{L_o}{T_o},$$

so the energy sector and the spacetime sector are not independent. Every change of the relational state rescales (E_o, M_o) and (T_o, L_o) coherently so that this equality is always preserved. The familiar practice of treating energy-mass and space-time as separate blocks is therefore an ontological approximation: in WILL they are locked by a single relational constraint.

Geometry \equiv Energy \equiv Causality \equiv WILL,

$$W_{ILL} = 1.$$

in the precise sense that one and the same conserved relational resource appears as mass, energy, time and length, but always in a way that keeps their ratio fixed.

16.2 Interpretive Note: The Name "WILL"

The term **WILL** stands for **SPACE-TIME-ENERGY**. It is both a formal shorthand and a philosophical statement: the universe is not a stage where energy acts through time upon space, but a single self-balancing structure whose internal distinctions generate all phenomena. The name also serves as a gentle irony toward anthropic thinking: the Cosmos does not possess "will" - yet through WILL, it manifests All that Is.

Summary

$$\mathbf{WILL} \equiv \frac{ET}{ML} = 1 \quad \iff \quad \text{Geometry} = \text{Energy} = \text{Causality.}$$

WILL is not the unit of something - but the Unity of Everything.

17 Ontological Shift: From Descriptive to Generative Physics

In conventional physics the methodology follows a descriptive paradigm:

1. Observable phenomena are identified.
2. Empirical regularities are codified as “laws of nature.”
3. Mathematical formalisms are constructed to *describe* these regularities.

Thus, physical laws are always introduced as external assumptions that model what is seen. Even in General Relativity, where geometry plays the central role, the equivalence principle and the metric postulate are still external inputs.

The RG framework inverts this paradigm. Laws are not added on top of observations; they are *generated* as inevitable consequences of relational geometry:

- There are no independent axioms such as “inertial mass equals gravitational mass.”
- Such relations appear automatically as algebraic identities enforced by the geometry.
- What classical physics calls “laws of nature” are secondary shadows of the single relational principle:

$$\text{SPACETIME} \equiv \text{ENERGY.}$$

Summary

Standard Physics: Laws *describe* what we observe.

Relational Geometry: Laws are *generated* as necessary products of closure and self-consistency.

In this sense, the ontological role of physical law is transformed. Physics ceases to be a catalog of empirical descriptions, and becomes the logical unfolding of a single relational structure. WILL identifies the necessary conditions under which all observed phenomena arise.

Descriptive Physics (Standard)	Generative Physics (WILL)
Phenomena are <i>observed</i> first, then summarized into empirical laws.	Laws emerge as <i>inevitable consequences</i> of relational geometry.
Physical laws are <i>assumptions</i> introduced to model reality.	Physical laws are <i>identities</i> , enforced by geometric self-consistency.
Time and space are treated as external backgrounds.	Time and space are <i>projections of energy relations</i> .
Dynamics = evolution of states <i>in time</i> .	Dynamics = ordered succession of balanced configurations; <i>time is emergent</i> .
Goal: <i>describe</i> what is observed.	Goal: <i>show why nothing else is possible</i> .

Table 4: Ontological contrast between standard descriptive physics and the generative paradigm of WILL Relational Geometry.

Phenomenon	Standard (GR) Result	Relational Geometry (RG)
GPS time shift / gravitational redshift	Frequency shift = combination of kinetic (SR) and gravitational (GR) effects.	Single symmetric law: $\tau = \beta_Y \cdot \kappa_X$, $E_{loc} = \frac{E_0}{\sqrt{(1-\beta^2)(1-\kappa^2)}} = \frac{E_{loc}}{\tau} 38.52$ $\mu\text{s/day}$ verified directly with GPS satellites.
Photon sphere, ISCO, horizons	Derived by solving geodesic equations in Schwarzschild metric.	Critical radii emerge from simple symmetry's (Photon sphere: $\theta_1 = \theta_2 = 54.73^\circ$ ("magic angle") $Q^2 = \kappa^2 + \beta^2 = 1$. ISCO: $Q = Q_Y$,
Mercury's perihelion precession	Complex expansion of Einstein field equations.	Same number obtained from RG with $\Delta\varphi = \frac{2\pi \cdot \tau_Y}{1-e^2} = 43''/\text{century}$. Using simple algebra.
Binary pulsar orbital decay	Explained via quadrupole radiation formula; requires asymptotic Bondi mass.	Emerges from balance of projection invariants without asymptotic constructs. $\Delta P \approx -2.42 \times 10^{-12}$ s/s (predicted)
Cosmological redshift	Photon "loses energy" as universe expands.	Energy conserved; redshift = redistribution of projection parameters. (Details in WILL PART II)
Cosmological absolute scale (Supernovae fit)	Hubble-like expansion, Λ CDM fits	$H_0 \equiv \sqrt{8\pi G \rho_\gamma / (3\alpha^2)} = 68.15$ Derived from CMB temperature and α connecting micro and macro scales (Details in WILL PART II-III).
Cosmological constant Λ	Added by hand to fit data ("dark energy").	Arises naturally as $\Lambda = 2/3r^2$. No extra entities required. (More details in WILL PART I and II)
Singularities	Predicted in black holes and big bang ($\rho \rightarrow \infty$).	Forbidden: density bounded by $\rho_{max} = c^2 / (8\pi G r^2)$.
Local gravitational energy	"Cannot be localized" (only ADM/Bondi at infinity).	Directly measurable via κ , e.g. from light deflection angle or red shift.
Unification with QM	No natural unification in GR framework.	Same projectional law applies from microscopic $\alpha = \beta_1$ (QM) to cosmic $\kappa^2 = \Omega_\Lambda$ (GR, COSMO) scales. (Details in WILL PART II and III)

Table 5: Classical GR results vs. WILL RG outcomes. Known effects are recovered by simpler symmetric laws, while new predictions eliminate singularities and explain cosmology without dark entities.

18 Conclusion

WILL Relational Geometry fully reproduces the predictive content and central equations of both Special and General Relativity, while simultaneously addressing their foundational inconsistencies:

- (1) the lack of an operational definition of local gravitational energy density in GR,
- (2) the artificial separation of kinetic and gravitational energy in SR and GR, and
- (3) the emergence of singularities as pathological artifacts of coordinate-based models. By treating energy and its transformations as the true basis of geometry, RG unifies and extends these frameworks into a fully relational and operationally grounded description of spacetime and energy.

By focusing on the projectional nature of energy, we have shown that spacetime itself is merely the manifestation of energy.

From a single Ontological Principle-that spacetime is equivalent to energy - we derived all the mathematical apparatus needed to describe gravitational and relativistic phenomena. This unification, showing that energy, time and space are merely different projections of the same underlying structure.

Special and General Relativity emerge from the same geometric principles.

This approach offers distinct advantages:

- Conceptual clarity - understanding physics through pure geometry
- Computational efficiency - significantly reducing complexity
- Epistemological hygiene - deriving results from minimal assumptions
- Philosophical depth - redefining our understanding of time, mass, and causality

WILL Relational Geometry inverts our fundamental understanding:

Spacetime and energy are mutually defining aspects of a single relational structure.

Final Summary

SPACETIME \equiv ENERGY.

19 References:

References

- [1] Schwarzschild, K. (1916). *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften*.
- [2] Tolman, R.C. (1939). *Phys. Rev.* **55**, 364; Oppenheimer, J.R., Volkoff, G.M. (1939). *Phys. Rev.* **55**, 374.
- [3] Dyson, F.W., Eddington, A.S., Davidson, C. (1920). *Phil. Trans. R. Soc. A* **220**, 291-333.
- [4] Lebach, D.E. et al. (1995). *Phys. Rev. Lett.* **75**, 1439.
- [5] Ashby, N. (2003). *Living Rev. Relativity* **6**:1.
- [6] Will, C.M. (2014). *Living Rev. Relativity* **17**:4.
- [7] Williams, J.G., Turyshev, S.G., Boggs, D.H. (2006). *Adv. Space Res.* **37**, 67-71.
- [8] Kramer, M. et al. (2021). *Phys. Rev. X* **11**, 041050.
- [9] Misner, Thorne, Wheeler (1973). *Gravitation*.
- [10] et al 2022 *ApJ* 933 49 DOI 10.3847/1538-4357/ac752f
- [11] Espenak, F. (2014). Seven Century Catalog of Mercury Transits: 2001 to 2700. *NASA Eclipse Web Site*.
- [12] Williams, M. (2016). How Far is Mercury From the Sun? *Universe Today* (citing NASA data).
- [13] Mamajek, E.E. et al. (2015). IAU 2015 Resolution B3 on Recommended Nominal Conversion Constants. *arXiv:1510.07674*.
- [14] Meftah, M., et al. (2014). *Astron. Astrophys.* **569**, A15.
- [15] Williams, D.R. (2024). *NASA Planetary Fact Sheet*. Goddard Space Flight Center. <https://nasa.gov>
- [16] Folkner, W.M., et al. (2014). *IPN Progress Report* **42-196**, 1-81.
- [17] Gonzalez Hernandez, J.I., et al. (2020). *Astron. Astrophys.* **643**, A146.
- [18] NASA Radio JOVE Project, *Jupiter Fact Sheet*. <https://radiojove.gsfc.nasa.gov/education/jupiter/basics/jfacts.htm>, 13 July 2000.