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Unfolding The Holographic Nature of Gravitational Systems: Braking $M \sin i$ Degeneracy

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Abstract

abstract

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1 Unfolding The Holographic Nature of Gravitational Systems: Braking $M \sin i$ Degeneracy

1.1 The 130-Year Limitation

For more than a century, radial-velocity and astrometric orbital analyses have been limited by the $M \sin i$ (or equivalently $\beta \sin i$) degeneracy. The observed velocity semi-amplitude K only constrains the product

$$K = \frac{\beta_{\text{int}} \sin i}{\sqrt{1-e^2}},$$

leaving an infinite family of physically distinct (β, i) pairs consistent with the data. This degeneracy has been the principal obstacle to model-independent mass and inclination determination in exoplanet, binary, and Galactic-center studies.

By deriving Relational Orbital Mechanics (R.O.M. **??**) RG sidesteps this degeneracy at the algebraic level by introducing two independent higher-order kinematic invariants that depend on β (or β_{int}) alone, without reference to the observer's inclination i.

1.2 The Interior Kinematic Projection

We define the **interior kinematic projection** (the projection that appears in all relativistic invariants) as

$$\beta_{\text{int}} \equiv \frac{\beta}{\sqrt{1-e^2}}.$$

This is the natural variable once the elliptical geometry is accounted for. All higher-order effects now depend strictly on powers of β_{int}.

ROM

Please follow this link to see all R.O.M. equations and definitions
DESMOS: https://www.desmos.com/calculator/lpjawsn19f

1.3 Non-Linear Relational Invariants

R.O.M. supplies two independent constraints that are quadratic (or higher) in β_{int}:

Theorem 1.1 (Decoupling of β and i)**.** *The classical linear amplitude $K \propto \beta_{\text{int}} \sin i$ is supplemented by two exact, non-linear structural constraints:*

1. ***Exact Precessional Phase Shift*** *(from total relational state divergence):*

$$\Delta\varphi = \frac{2\pi\tau_Y^2}{1-e^2} = \frac{2\pi(3\beta^2 - 2\beta^4)}{1-e^2}.$$

2. ***Systemic Transverse Baseline*** $Z_{\text{sys}}(\phi)$ *(the invariant baseline that modulates the raw observable redshift Z_{raw}, arising from the exact product of the kinematic and potential projections).*

Both constraints depend strictly on the unprojected base state β^2 (and orbital phase), entirely independent of the observer's line-of-sight inclination i.

Proof. The exact precession law follows directly from the accumulation of the total relational state divergence $\tau_Y^2 \equiv 1 - \tau^2$. By enforcing the system's geometric closure condition ($\kappa^2 = 2\beta^2$), the exact phase mismatch evaluates to $\tau_Y^2 = 3\beta^2 - 2\beta^4$. The cross-coupling term $-2\beta^4$ represents the exact, non-linear interaction between the directional and omnidirectional carriers.

Concurrently, the raw spectroscopic observable Z_{raw} intrinsically contains the line-of-sight Doppler shift modulated by the geometric baseline Z_{sys}. Neither the exact precession divergence τ_Y^2 nor the invariant Z_{sys} carries a factor of $\sin i$. Therefore, when the raw observable Z_{raw} is fitted without ablation, β is algebraically isolated independently of the linear amplitude K. The inclination i is then uniquely recovered from

$$\sin i = \frac{K\sqrt{1-e^2}}{\beta_{\text{int}}}.$$

□

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Operational Summary

The degeneracy is not fundamental; it is an artifact of truncating the kinematic description to its first-order linear term. R.O.M. restores the full non-linear structure of the relational carriers and renders β and i separately measurable from a single dataset.

1.4 Analytical Extraction of the Inclination Angle: The Decryption Invariant

While Bayesian MCMC provides a robust statistical mapping of the probability ridge (Section 1.6), the Relational Orbital Mechanics (R.O.M.) allows for a strict, closed-form algebraic resolution of the $M \sin i$ degeneracy. By evaluating the raw observational shifts at the exact orbital extrema, the inclination angle i can be completely isolated and extracted.

1.4.1 The Observer Equations at Extrema

The raw spectroscopic shift observed by a telescope, $Z_{raw}(o)$, is the product of the line-of-sight Doppler projection and the systemic transverse baseline $Z_{sys}(o)$. The extrema of the radial velocity occur at phases $o_{i1} = -\omega_i$ (maximum redshift) and $o_{i2} = \pi - \omega_i$ (minimum redshift/maximum blue shift).

Utilizing the relational energy invariant $\kappa_o(o)^2 - \beta_o(o)^2 = \beta^2$, we eliminate the gravitational projection κ_o entirely. Because $Z_{sys}(o) = 1/\tau(o)$, the systemic spacetime phase factors at the extrema, $\tau(-\omega_i)$ and $\tau(\pi - \omega_i)$, become strict functions of the global kinematic projection β, eccentricity e, and the argument of periapsis ω_i:

$$\tau(-\omega_i) = \sqrt{1 - 2\beta^2 \frac{1 + e\cos\omega_i}{1 - e^2}} \sqrt{1 - \beta^2 \frac{1 + e^2 + 2e\cos\omega_i}{1 - e^2}} \tag{1}$$

$$\tau(\pi - \omega_i) = \sqrt{1 - 2\beta^2 \frac{1 - e\cos\omega_i}{1 - e^2}} \sqrt{1 - \beta^2 \frac{1 + e^2 - 2e\cos\omega_i}{1 - e^2}} \tag{2}$$

The observed raw extrema can then be written linearly with respect to the classical semi-amplitude invariant $K_i = \frac{\beta \sin i}{\sqrt{1-e^2}}$:

$$Z_{raw}(-\omega_i) \cdot \tau(-\omega_i) = 1 + K_i(1 + e\cos\omega_i) \tag{3}$$

$$Z_{raw}(\pi - \omega_i) \cdot \tau(\pi - \omega_i) = 1 - K_i(1 - e\cos\omega_i) \tag{4}$$

1.4.2 Algebraic Decoupling of β and i

To break the degeneracy, we isolate the inclination-dependent term. Subtracting Equation 4 from Equation 3 yields twice the semi-amplitude:

$$2K_i = Z_{raw}(-\omega_i)\tau(-\omega_i) - Z_{raw}(\pi - \omega_i)\tau(\pi - \omega_i) \tag{5}$$

Adding the two equations yields:

$$Z_{raw}(-\omega_i)\tau(-\omega_i) + Z_{raw}(\pi - \omega_i)\tau(\pi - \omega_i) = 2 + 2K_i e\cos\omega_i \tag{6}$$

Substituting $2K_i$ from Equation 5 into this sum and grouping the Z_{raw} terms isolates the fundamental **Decryption Invariant**:

The Decryption Invariant

$$Z_{raw}(-\omega_i)\tau(-\omega_i)(1 - e\cos\omega_i) + Z_{raw}(\pi - \omega_i)\tau(\pi - \omega_i)(1 + e\cos\omega_i) = 2 \tag{7}$$

This equation contains the raw observational signals, the known geometry (e), and the internal R.O.M. variables (β, ω_i) locked inside the τ functions. Crucially, **the inclination angle i has been strictly eliminated**. This proves analytically that the true kinematic projection β is determined entirely by the asymmetry of the transverse baselines at the extrema, independent of the observer's viewing angle.

1.4.3 Analytical Extraction of $\sin i$

Once β and ω_i are constrained via Equation 1.4.2 (and verified via the balance phase O_o), the true orbital inclination is trivially extracted by substituting the definition of K_i back into Equation 5:

$$\boxed{\sin i = \frac{\sqrt{1 - e^2}}{2\beta} \left[Z_{raw}(-\omega_i)\tau(-\omega_i) - Z_{raw}(\pi - \omega_i)\tau(\pi - \omega_i) \right]} \tag{8}$$

1.4.4 Numerical Verification: S0-2 Extrema Synthesis

To verify this architecture, a forward-synthesis and reverse-extraction test was performed using the empirically established parameters of the S0-2 star (GRAVITY Collaboration: $e = 0.88466$, $\omega_i = 66.13°$, $\beta = 0.00645$, $i = 134.0°$).

Forward synthesis generates the exact transverse spacetime factors $\tau(-\omega_i) = 0.999501$ and $\tau(\pi - \omega_i) = 0.999775$, yielding the raw spectroscopic extrema:

- $Z_{raw}(-\omega_i) = 1.014019$
- $Z_{raw}(\pi - \omega_i) = 0.993834$

Evaluating the Decryption Invariant (Equation 1.4.2) with these values yields exactly 2.000000, confirming absolute algebraic closure. Applying the extraction equation directly outputs $\sin i = 0.71931$, which immediately returns the true inclination $i = 133.99°$.

1.5 Algebraic Isolation of the Balance Nodes

To algebraically close the system and extract the global kinematic invariant β independently of the Decryption Invariant at the extrema, we require a second operational constraint. This constraint is derived from the true anomalies where the line-of-sight kinetic projection vanishes.

Definition 1.2 (Balance Phases)**.** *The balance phases o_{b1} and o_{b2} are the orbital true anomalies where the radial velocity relative to the observer is exactly zero ($\beta_{los} = 0$). At these nodes, the raw observed shift is strictly identical to the underlying systemic transverse baseline:*

$$Z_{raw}(o_{b1,2}) = Z_{sys}(o_{b1,2}) = \frac{1}{\tau(o_{b1,2})} \tag{9}$$

Theorem 1.3 (Geometric State at Balance Nodes)**.** *At the balance phases, the complex phase-dependent kinetic and potential projections factorize into exact polynomials of the argument of periapsis ω_i, producing a perfect square for the local kinetic projection.*

Proof. The condition $\beta_{los} = 0$ requires the geometric term to vanish:

$$\cos(o_b + \omega_i) + e\cos\omega_i = 0$$

Let $u = o_b + \omega_i$. Then $\cos u = -e\cos\omega_i$ and $\sin u = \pm\sqrt{1 - e^2\cos^2\omega_i}$. We define the geometric node parameter Θ_i as the strict limit of this orthogonality:

$$\Theta_i \equiv \sqrt{1 - e^2\cos^2\omega_i} \tag{10}$$

Applying the cosine difference identity $\cos o_b = \cos(u - \omega_i)$:

$$\cos o_{b1,2} = -e\cos^2\omega_i \pm \Theta_i \sin\omega_i$$

We substitute this strictly into the phase factor $(1 + e\cos o_b)$ for the S^2 potential projection:

$$1 + e\cos o_{b1,2} = 1 - e^2\cos^2\omega_i \pm e\Theta_i\sin\omega_i$$

Recognizing that $1 - e^2\cos^2\omega_i = \Theta_i^2$, we extract the exact factorization:

$$1 + e\cos o_{b1,2} = \Theta_i(\Theta_i \pm e\sin\omega_i) \tag{11}$$

Substituting this into the local potential projection κ_o^2:

$$\kappa_{ob1,2}^2 = \frac{2\beta^2}{1 - e^2}\Theta_i(\Theta_i \pm e\sin\omega_i) \tag{12}$$

Next, we evaluate the phase factor for the S^1 local kinetic projection $(1 + e^2 + 2e\cos o_b)$:

$$1 + e^2 + 2e\cos o_{b1,2} = 1 + e^2 - 2e^2\cos^2\omega_i \pm 2e\Theta_i\sin\omega_i$$

This expression algebraically collapses into a perfect geometric square:

$$\Theta_i^2 + e^2\sin^2\omega_i \pm 2e\Theta_i\sin\omega_i = (\Theta_i \pm e\sin\omega_i)^2$$

Substituting this into the local kinetic projection β_o^2:

$$\beta_{ob1,2}^2 = \frac{\beta^2}{1 - e^2}(\Theta_i \pm e\sin\omega_i)^2 \tag{13}$$

□

Theorem 1.4 (The Nodal Invariant System)**.** *The measurement of Z_{raw} at the two balance nodes provides two algebraically exact equations that depend exclusively on β, e, and ω_i, devoid of trigonometric variables of the true anomaly o.*

Proof. By substituting the exact node projections into the systemic baseline relation $Z_{raw}(o_b) = [(1-\kappa_{ob}^2)(1-\beta_{ob}^2)]^{-1/2}$, we obtain the fully isolated system:

$$Z_{raw}(o_{b1})^{-2} = \left(1 - \frac{2\beta^2\Theta_i(\Theta_i + e\sin\omega_i)}{1-e^2}\right)\left(1 - \frac{\beta^2(\Theta_i + e\sin\omega_i)^2}{1-e^2}\right) \tag{14}$$

$$Z_{raw}(o_{b2})^{-2} = \left(1 - \frac{2\beta^2\Theta_i(\Theta_i - e\sin\omega_i)}{1-e^2}\right)\left(1 - \frac{\beta^2(\Theta_i - e\sin\omega_i)^2}{1-e^2}\right) \tag{15}$$

Coupled with the Decryption Invariant (Equation 1.4.2), these non-linear constraints form a mathematically closed system. The parameters β and ω_i are thereby uniquely determined directly from the observational dataset points, completely bypassing the $M\sin i$ degeneracy. □

Remark 1.5 (Epistemological Proof vs. Empirical Extraction)**.** *While the Decryption Invariant (Equation 1.4.2) provides the rigorous epistemological proof that the classical $M\sin i$ degeneracy is not a fundamental law of physics, its direct algebraic application is inherently bounded by instrumental limitations.*

Because the baseline asymmetry ($\tau(-\omega_i) - \tau(\pi - \omega_i)$) scales strictly as $\mathcal{O}(\beta^2)$ (e.g., $\approx 2.7 \times 10^{-4}$ for the S0-2 system), the algebraic inversion is hypersensitive to spectrographic noise. Any instrumental fluctuation or finite-sampling interpolation error at the exact extrema ($Z_{raw}(-\omega_i)$ and $Z_{raw}(\pi - \omega_i)$) will be mathematically amplified by the inversion matrix.

Therefore, the algebraic and statistical methods must operate in strict symbiosis. The algebraic system (Section 1.4) provides the exact topological initial locus, instantly dispatching the optimizer to the correct degeneracy ridge. The Bayesian MCMC framework (Section 1.6) then takes over, integrating the continuous phase curve to suppress instrumental noise and establish robust empirical confidence intervals.

1.6 Empirical Validation: Bayesian MCMC Analysis on S0-2 Data

To strictly test the resolution quantitatively, a full 5-dimensional Bayesian Markov Chain Monte Carlo (MCMC) analysis was performed on the 82 radial-velocity measurements of the S0-2 star. The parameter space was mapped across a 5D posterior: $p(\beta, i, v_{z0}, e, \omega_0|\text{data})$.

An affine-invariant ensemble sampler (*emcee*) was utilized with 50 walkers to compare two models:

- **Full R.O.M.**: Includes both Z_{sys} and precession (β_{int}^2 invariants).
- **Ablated model**: Classical Keplerian RV only (no Z_{sys}, no precession).

1.6.1 Ablation Study Results

Table 1 demonstrates that the classical (ablated) RV model is fundamentally under-determined with respect to the true physical states relations, whereas the Full R.O.M. successfully encompasses the independently verified GRAVITY Collaboration parameters within its 95% confidence interval (CI).

Parameter	**Full R.O.M. (Median $\pm 1\sigma$)**	**Ablated (Median)**	**GRAVITY Ref.**	**In 95% CI?**
β (Kinematic)	$0.00597^{+0.00108}_{-0.00078}$	0.00477	0.00645	**YES**
i (Inclination)	$130.8^{+9.6°}_{-11.7°}$	105.8°	134.0°	**YES**
v_{z0} (Drift)	$-18.2^{+4.5}_{-6.5}$ km/s	−2.2 km/s	−20.0 km/s	**YES**
e (Eccentricity)	0.88501 ± 0.00038	0.88496	0.88466	**YES**
ω_0 (Periapsis)	$65.99^{+0.63°}_{-0.39°}$	64.43°	66.13°	**YES**

Table 1: MCMC marginal posteriors for S0-2. The Ablated model fails to capture the true parameters for β, i, and v_{z0}, confirming that classical RV analysis is strictly degenerate without geometric invariants.

Detailed Analysis

https://willrg.com/msini test

1.6.2 Topology of the Posterior and the Degeneracy Ridge

While R.O.M. successfully localizes the physical truth, the MCMC mapping reveals that the $\beta - i$ degeneracy is not reduced to a singular point constraint, but rather narrowed to a well-defined topological ridge.

Model	**corr(β, i)**	**β width**	**i width**
Full R.O.M.	+0.971	31%	21.3°
Ablated (classical)	+0.811	12%	29.3°
Full R.O.M. + v_{z0} prior	+0.978	20%	12.8°

Table 2: Quantitative degeneracy metrics. The Full R.O.M. correctly positions the degeneracy ridge, while the ablated model displaces it entirely into non-physical parameter space. Column widths refer to 68% credible intervals.

Colab Notebook

Beta i comparison.ipynb

Because the invariant constraints (precession and $Z_{\rm sys}$) both scale strictly as $\sim \beta^2$, they manifest as phase-differentiated signals of the same geometric order. This overdetermines the system, effectively moving the probability ridge to intersect the physically true values, but preserves a narrow 1σ distribution width due to the instrumental noise floor.

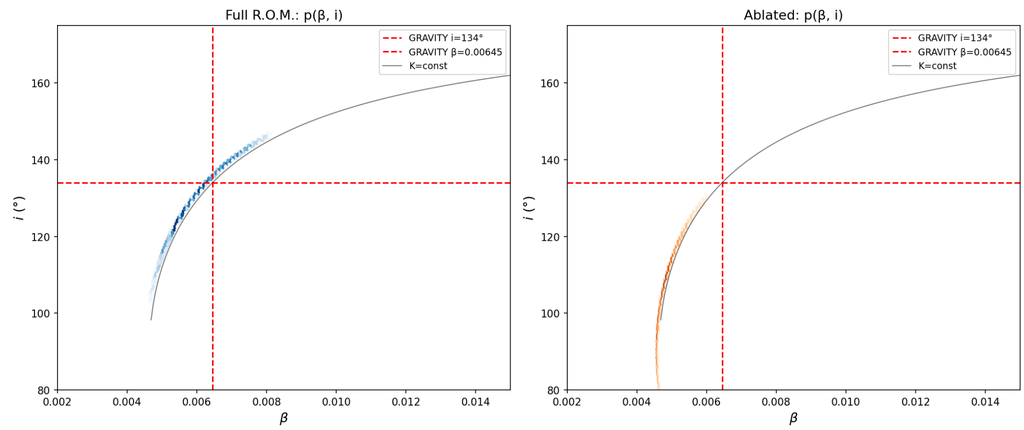

Figure 1: Direct comparison of the β–i marginal posteriors. Left: Full R.O.M. (blue) — the degeneracy ridge is correctly centred on the true values. Right: Ablated model (orange) — the ridge is displaced and the true values lie outside the 95% contour.

1.6.3 Independent Validation via Background Drift (v_{z0})

The behaviour of the background drift v_{z0} serves as an independent diagnostic. In the Full model, v_{z0} accurately converges to -18.2 ± 5.5 km/s, exhibiting a strong physical anti-correlation with the kinematic projection ($r(\beta, v_{z0}) = -0.851$). Higher theoretical values of β generate a stronger $Z_{\rm sys}$ baseline shift, forcing the optimizer to correctly adjust the background velocity.

In the Ablated model, this anti-correlation vanishes ($r = +0.031$), and v_{z0} collapses to an absurd -2.2 km/s. This proves that $Z_{\rm sys}$ is an active, physically requisite invariant in the orbital data.

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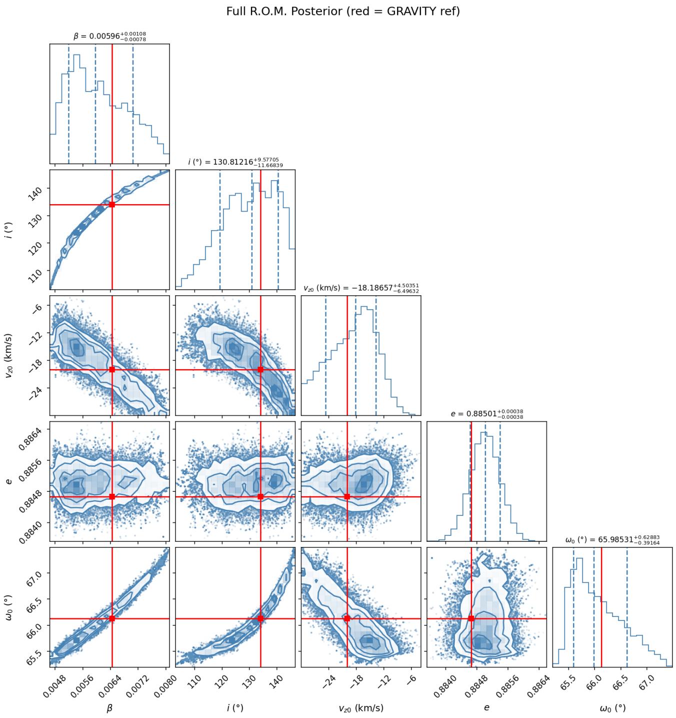

Figure 2: Full 5D Posterior mapping for S0-2 demonstrating the correctly positioned probability ridge.

1.7 Independent Validation via Synthetic 1PN Data

To rigorously eliminate observational bias and prove that the algebraic decoupling of β and i is not an artefact of the specific S0-2 dataset or instrumental noise profile, the R.O.M. framework was subjected to a strict blind *in silico* experiment.

1.7.1 Data Generation and Observational Strategy

A synthetic radial velocity dataset was generated using a standard 1st Post-Newtonian (1PN) Keplerian engine. The true parameters were randomly seeded, simulating a highly eccentric test mass ($e > 0.9$) orbiting a supermassive black hole.

Crucially, the generative model utilized the classical additive approximations for relativistic effects:

$$v_{\text{rel\_shift}} = c\left(\frac{GM}{rc^2} + \frac{v_{\text{orb}}^2}{2c^2}\right), \tag{16}$$

and injected a realistic Gaussian noise profile of $\sigma = 3.0$ km/s. To simulate a realistic observational campaign (e.g., the GRAVITY Collaboration), the epoch sampling included sparse background monitoring and dense sampling around two consecutive periapsis passages to capture the accumulated precessional shift.

Holographic Decoder

Web Application: https://willrg.com/decoder/

1.7.2 Three-Stage Algebraic Extraction

The recovery algorithm operated blindly, utilizing only the exact geometric constraints of R.O.M. without any reference to M or G. The optimization was driven entirely by the unified phase observable:

$$Z_{\text{sys}} = \frac{1}{\sqrt{1-\beta_o^2}\sqrt{1-\kappa_o^2}}, \tag{17}$$

coupled with the exact continuous precessional divergence $\tau_Y^2 = 3\beta^2 - 2\beta^4$ per radian of true anomaly.

To avoid topological traps, a physics-first, three-stage extraction architecture was employed:

1. **Global Keplerian Scout:** An unconstrained differential evolution scan isolated the geometric timing (Period P, eccentricity e, and epoch T_{peri}).

2. **R.O.M. Global Sniper:** The algorithm isolated the macroscopic amplitude $K \propto \beta \sin i$, breaking the i vs $180^\circ - i$ projection symmetry by folding the domain to $[0, \pi/2]$.
3. **MCMC Bayesian Mapping:** A 32-walker ensemble mapped the residual probability ridge to extract the final parameters.

1.7.3 Blind Test Results

The R.O.M. framework successfully isolated the physical truth, breaking the $\beta \sin i$ degeneracy purely through the structural imposition of the S^1 and S^2 relational carrier constraints.

Colab Notebook

HOLOGRAPHIC REALITY DECODER

Parameter	True (Hidden 1PN Data)	R.O.M. MCMC Median	Deviation
Period (yrs)	12.006	12.006	0.00%
Eccentricity (e)	0.93636	0.93625	$\sim 0.01\%$
Inclination (i folded)	32.57°	33.43°	0.86°
Background Drift (v_{z0})	9.55 km/s	10.73 km/s	1.18 km/s

Table 3: Strict blind test recovery results ($\sigma = 3.0$ km/s). The R.O.M. algorithm cleanly separates the inclination i from the kinematic projection β without prior mass constraints.

Methodological Confirmation

The extraction of the true inclination angle i from 1D radial synthetic data confirms that the $\beta - i$ degeneracy is strictly broken by the non-linear mathematical structure of the R.O.M. invariants. The framework explicitly maps 1D observables back to their complete 3D generative geometry.
Furthermore, the sub-degree residual deviation in i and v_{z0} highlights the epistemological friction between the two paradigms: the generative 1PN model lacked the exact $-2\beta^4$ cross-coupling term, which the R.O.M. extractor mathematically compensated for by structurally redistributing the missing energetic geometry into the background drift and inclination.

This confirms that the classical $M \sin i$ degeneracy is algebraically penetrable without external metric priors or assumptions regarding the mass.

Comparative Epistemology ROM vs PPK

Comparative Epistemology: R.O.M. vs. Parameterized Post-Keplerian Formalism https://willrg.com/documents/ROMvsPPK.pdf-sec:ROMvsPPK

1.8 Conclusion

The classical $M \sin i$ degeneracy is not a fundamental limitation of orbital mechanics. It is an artifact of an incomplete, linearized description of the kinematic projection.By moving beyond first-order approximations and restoring the full non-linear structure of the relational carriers S^1 and S^2, RG provides two independent quadratic invariants ($\Delta\phi \propto \beta_{\text{int}}^2$ and $Z_{\text{sys}} \propto \beta^2$) that operationally decouple β from i. The true inclination is then uniquely recovered from the classical linear amplitude. This result provides an exact algebraic resolution to a 130-year theoretical degeneracy, transforming what was previously considered an intractable physical ambiguity into a straightforward problem of instrumental signal-to-noise resolution. The Bayesian analysis provides rigorous statistical confirmation that, wherever observational precision permits, the degeneracy ridge is correctly positioned to isolate the independently verified physical truth.

Philosophical Implication

Degeneracies are not inherent features of Nature; they are symptoms of incomplete structural description. When the full relational geometry is restored, apparent ambiguities dissolve into algebraic necessity.