

WILL RG

Where do dimensionality and topology themselves come from relationally?

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Abstract

Hi Luke! I compiled this compressed derivation to specifically address your question and demonstrate how topology and degrees of freedom can be derived just from relational epistemic constraints. Here's the logical chain of this document: Operational realism + relationalism = 4 epistemic constraints $\rightarrow$ 3 core theorems $\rightarrow$ Unifying Principle $\rightarrow$ WILL Structure S^1 and S^2 . All of this is from the beginning of the first paper in the [WILL RG Trilogy](#). I add the chart of full derivation chain from the full first paper just so you could see where is it going.

I thought this format would be easier for you to read, and also I just so used to latex that its easier for me to write this way. I hope this is not too overwhelming...

1 Stage I: Ontology Emerges from Epistemology

Every logical step is subject to empirical audit: if any consequence of this chain contradicts observation, the chain fails.

Remark 1.1 (On mirroring effect). *As humans, we often project ourselves onto observed phenomena, obscuring the true nature of physical reality behind our anthropic tendencies. In order to prevent this mirroring effect, theoretical development has to establish strong epistemic constraints on scientific reasoning. Without this constraints our resulting theories risking telling us more about ourselves then about the nature of the Universe.*

1.1 Foundational Epistemic Principles

Flow chart of derivation logic

[Logos Map](#)

Principle 1.2 (Epistemic Hygiene). Refusal to import unjustified assumptions or arbitrary choices.

This line of reasoning derives physics by **removing hidden assumptions**, rather than introducing new postulates. This construction is deliberate and contains zero free parameters. This is not a simplification - it is a deliberate epistemic constraint. No assumptions are introduced and no constructs are retained unless they are geometrically or energetically necessary.

Principle 1.3 (Relational Origin). As default epistemic stance, all physical quantities defined by their relations, until and if their absolute nature is proven.

Any introduction of unproven absolute properties risks reintroducing metaphysical artefacts and contradicts the foundational insight of relationalism.

Principle 1.4 (Ontological Minimalism). *Maintain the minimum number of foundational primitives by peeling off interpretational layers until only irreducible core is left. Spatial container, axis of time, distance, forces, fields, intrinsic isolated properties — are excluded as preexisting primitives.*

The terms (time, distance, forces, fields) later appear as translation into common descriptive vocabulary.

Principle 1.5 (Mathematical Transparency). *Every mathematical expression, phrase, operation, or identity is treated as a binding ontological statement. This force's mathematics to always reflect direct physical meaning preventing obscure constructs. Every symbol must be anchored to unique physical idea with transparent ontological origin.*

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This principle transform mathematics from merely a toolbox we use the way we see fit, in to rigours language where every single element has to be ontologically justified by internal derivation or external direct empirical measurement. Introducing symbols without explicit necessity constitutes **Over-parameterization**: the proliferation of symbols without corresponding physical meaning. Number of symbols = Number of independent physical ideas.

Mathematics is a language, not a world. Its symbols must never outnumber the physical meanings they encode.

A Note on Methodology vs. Ontology

It may be objected that the foundational principles constitute axioms, thereby contradicting the claim of *zero assumptions*. This objection conflates *methodological constraints* with *ontological axioms*.

Under the strictures of the Scientific Method, the burden of proof rests exclusively on the claimant asserting an absolute existence. Refusing to import unobserved absolutes (such as a background container or intrinsic properties) is not an assumption; it is the default epistemic stance — the null hypothesis.

These principles function strictly as rules of logical engagement to prevent the introduction of surplus ontology. To challenge them is to argue for the admission of a specific unobserved absolute — and under the empirical foundation of physics, the burden of proof rests with that admission.

1.1.1 The Foundational Core: Relational Geometry

These epistemic principles lead us to the core of Relational Geometry:

Theorem 1.6 (Relational Closure). *A purely relational system cannot be geometrically open; any stable, self-contained system must be described by a closed and self-consistent relational geometry. Its geometry is closed by absolute analytic necessity.*

Proof by Analytic Tautology.

1. **Premise 1 (Relational Origin)**: Within RG Physical quantities are defined exclusively by their relations (Principle 1.3).
2. **Premise 2 (Interaction as Integration)**: If a hypothetical “outside” entity or boundary were to physically interact with the system, that interaction itself constitutes a relation.
3. **Deduction (The Tautology)**: As relation is formed, it is mathematically and physically integrated into the system.

An “open relational system” experiencing external influence is a logical contradiction. The system is closed not by a spatial boundary or a metric container, but by the strict impossibility of an interacting non-relation. The system is relationally closed by definition. □

1.1.2 What is Energy in Relational Ontology?

Across all domains of physics, one empirical fact persists: in every closed system there exists a quantity that never disappears or arises spontaneously, but only transforms in form. This invariant is observed under many guises — kinetic, potential, thermal, chemical — yet all are interchangeable, pointing to a single underlying structure.

Crucially, this quantity is never observed directly, but only through *differences between states*: a change of velocity, a shift in configuration, a transition of phase. Its value is relational, not absolute: it depends on the chosen frame or comparison, never on an object in isolation.

Moreover, this quantity provides continuity of causality. If it changes in one part of the system, a complementary change must occur elsewhere, ensuring the unbroken chain of transitions. Thus it is the bookkeeping of causality itself — the ledger of change.

Corollary 1.7 (Energy). *In classical and relativistic mechanics, this derived invariant measure of change is assigned the historical label “Energy”. By showing its relational origin, we strip “Energy” of its substantialist properties (as a “substance” or “thing”) and reveal its origin as the necessary measure of change and bookkeeping ledger of causality.*

Energy :

Definition 1.8 (Energy).

*Energy is the invariant relational measure
of state transformation
within closed system.*

Remark 1.9 (Energy Ledger: intuitive understanding). *Consider a completely closed economy with no physical currency — no dollar bills, no gold coins, and no central bank (acting as an absolute container). It is a purely relational economy consisting solely of a massive ledger of debts and credits between individuals. If one account decreases, another must increase. If the system is frozen at any given moment, the sum of all debts and credits across the entire ledger must equal zero. "Money" does not exist as a physical object; it is simply the mathematical rule that the ledger must always balance. Energy functions identically: it is not a substance, but the invariant rule of relational accounting.*

Theorem 1.10 (Causal Continuity (Conservation)). *Without background any closed dynamical system must conserve its total energy. Any energy change must be perfectly balanced by a complementary change elsewhere in the system enforcing Conservation and Causal Continuity.*

Proof by Structural Necessity.

1. **Premise 1 (Relational Origin):** Physical quantities are defined exclusively by their relations. No background allowed (Principle 1.3).
2. **Premise 2 (Empirical Change):** States undergo observable Change.
3. **Premise 3 (Relational Closure):** As proven in Theorem 1.6, the relational system is analytically closed. There is no external background, void, or hidden container to absorb or inject influence.
4. **Deduction (Causal Continuity):** Because the system is closed (Premise 3), and no background allowed (Premise 1), an empirical change (Premise 2) cannot vanish into a void or emerge from one. To maintain the structural integrity of the relations (Premise 1), any change must be perfectly balanced by a complementary change elsewhere in the system. This enforcing **Conservation and Causal Continuity**.

□

Theorem 1.11 (Relational Isotropy). *Within a background-free relational system, no spatial direction or coordinate can be a priori privileged. Therefore, the geometry encoding the relational balance (energy) must be maximally symmetric (isotropic).*

Proof by Relational Origin.

1. **Premise 1 (Absence of a Grid):** As established by Theorem 1.6 (Relational Closure), the relational system possesses no external container, absolute metric tensor, or underlying coordinate grid.
2. **Premise 2 (Direction requires Relation):** Under the Principle of Relational Origin (Principle 1.3), “direction” or “orientation” cannot exist in a void; it only physically manifests as a specific interaction or measurement between defined participants.
3. **Deduction (Pure Gauge):** Because there is no absolute grid (Premise 1), assigning an intrinsic, absolute axis to the unmeasured relational ledger (energy) introduces unobservable “surplus ontology.” Any such arbitrary orientation is a mathematical artifact (pure gauge).

Therefore, prior to a specific interaction defining a local axis, the pure relational structure encoding this conserved energy ledger must possess no intrinsic privileged direction. It must be strictly isotropic. □

1.2 Unifying Ontological Principle

Summary

Any attempt to treat “*spacetime structure*” as separate from “*dynamics*” quietly imports a background container that is not justified by the phenomena. This violates epistemic hygiene: it introduces an ontological artifact without necessity. Eliminating this separation compels the identification of structure and dynamics as two aspects of a single entity.

Lemma 1.12 (False Separation). *Any model that treats processes as unfolding within an independent background necessarily assigns to that background structural features (metric, orientation, or frame) not derivable from the relations among the processes themselves. Such a background constitutes an extraneous absolute.*

Proof. Suppose an independent background exists. Then at least one of its structural attributes - metric relations, a preferred orientation, or a class of inertial frames - remains fixed regardless of inter-process data. This attribute is not relationally inferred but posited a priori. It thereby violates the relational closure principle: it introduces a non-relational absolute external to the system. Hence the separation is unwarranted. □

Corollary 1.13 (Structure–Dynamics Coincidence). *Without introducing new primitives 1.12, we should see structure and dynamics as a single ontological element. The structural arena and the dynamical content must be identified: structure is dynamics, energy is spacetime.*

This leads us to a principle with *negative ontological weight*: it reduces the number of primitives and removes the separation between structure and dynamics. Spacetime is not a "container" but relational tension between energy states.

Principle 1.14 (Unifying Ontological Principle).

$$\boxed{SPACETIME \equiv ENERGY}$$

No background container permitted.

Definition 1.15 (WILL). **WILL** $\equiv$ **SPACE-TIME-ENERGY** is the technical term we use for the unified relational structure determined by Unifying Principle 1.14. All physically meaningful quantities are relational features of WILL.

The Last Geocentric Epicycle

Definition 1.16 (Ontological weight). The number of irreducible primitives a theory posits.

Picture	Primitives
Substantial: container + content (spacetime, energy/matter)	≥ 2
Relational: WILL (one self-relating structure)	1

It is the operation physics has performed at every unification:

- **Heat** $\equiv$ molecular motion (deletes caloric);
- **Electricity** $\equiv$ magnetism (Maxwell);
- **Space** $\equiv$ time (Einstein).

Each merged two primitives into one. **SPACETIME** $\equiv$ **ENERGY** applies the identical operation. **This Principle does not add, it subtracts: it removes the hidden assumption. Structure and dynamics are two aspects of a single entity that we call WILL.**

Remark 1.17 (Auditability). Principle 1.14 is foundational but testable: it is subject to: (i) geometric audit (internal logical consequences) and (ii) empirical audit (agreement with empirical data).

2 Stage II: Geometric Derivation

2.1 WILL Structure

Having established Unifying Principle 1.14 by removing the unwarranted separation of structure and dynamics, we are left with a single foundational primitive: **WILL** $\equiv$ **SPACE-TIME-ENERGY**. Now we proceed to derive the necessary properties of **WILL**.

The foundational core (1.1.1) — Closure, Causal Continuity, and Isotropy — establishes the necessary geometric properties for the relational energy carriers.

2.2 Deriving Relational Carriers

Theorem 2.1 (Minimal Relational Carriers of the Conserved Relational Ledger). The minimal relational carriers of the same conserved energy ledger satisfying methodological constraints (1.1) and foundational core (1.1.1) of Relational Closure, Conservation, Maximal Symmetry Theorems are:

- S^1 for the minimal 1-DOF directional (Kinematic) relational Change;
- S^2 for the minimal 2-DOF omnidirectional (Potential) relational Change.

Proof. The carriers are determined by the minimum irreducible DOF required to represent observable relational Change, with no assumption of additional DOF beyond what observation requires.

1. 1-DOF (Kinematic) relational Change.

The simplest non-trivial physically distinguishable Change is a difference between two relational states, A and B . Its smallest non-zero irreducible relational structure has one degree of freedom.

This 1-DOF Change can be represented as a closed line or loop in relational state space. The minimal closed, maximally symmetric carrier is therefore the circle S^1 . The total Change ledger on S^1 we normalise to unity making S^1 - unit circle. Relational state correspond to the point on S^1 we call θ_1 with two orthogonal external and internal projections represent two aspects of the same 1-DOF relational Change:

$$\boxed{\beta^2 + \beta_Y^2 = 1}.$$

The projections β (external amplitude) and β_Y (internal phase) do not constitute two independent DOF; they are orthogonal projections of the same single relational degree of freedom.

The carrier is therefore:

- **Closed** (by Relational Closure Theorem 1.6 — no boundary)
- **Maximally symmetric** (by Isotropy Theorem 1.11 — no privileged point or direction on the carrier)

A linear segment cannot carry this relation because it possesses absolute endpoints. Endpoints act as boundaries where the transformational ledger would either vanish (violating the Conservation Theorem) or require an external bounding mechanism to reflect it (violating the Relational Origin and Ontological Minimalism principles 1.4, 1.3).

The unique 1-DOF boundaryless, isotropic carrier is the circle (S^1)

2. 2-DOF (Potential) relational Change.

The next irreducible relational structure arises when a central state A is related by the same Change to N equivalent states B_i , with no privileged direction among those relations. The minimum independent relational structure required to represent this isotropic omnidirectional distribution is 2 DOF.

The 2-DOF carrier must be:

- **Closed** (by Relational Closure Theorem 1.6 — compact, no boundary, no leakage into an external void.)
- **Maximally symmetric** (by Isotropy Theorem 1.11 — isotropic distribution of the relational Change.)
- **Orientable** (by Causal Continuity Theorem 1.10 — opposite directions are physically distinct, not identified)

Under these conditions, the minimal boundaryless, maximally symmetric 2-DOF carrier is the 2-sphere (S^2). Anisotropic surfaces such as the torus T^2 introduce additional directional structure, while the non-orientable $\mathbb{RP}^2$ identifies opposite orientations and therefore does not satisfy foundational core (1.1.1).

Meridional Section of S^2

By Isotropy Theorem 1.11, any radially symmetric relation reduces to a great-circle meridional section. The S^2 carrier remains the physical/relational carrier; the calculation is restricted to a symmetry-equivalent meridional section without loss of generality.

Same as S^1 the total relational Change ledger on S^2 we normalise to unity making S^2 a unit two-sphere. Relational state correspond to the point on meridional section of S^2 we call θ_2 with two orthogonal external and internal projections represent two aspects of the same 2-DOF relational Change:

$$\kappa_X^2 + \kappa^2 = 1.$$

The projections κ_X (internal phase) and κ (external amplitude) are not separate physical quantities; they are orthogonal projections of the same 2-DOF relational Change.

The unique boundaryless, maximally symmetric 2-DOF carrier is the 2-sphere (S^2)

3. Higher-Dimensional Carriers (S^n for $n \geq 3$).

The existence of mathematical carriers with $n \geq 3$ DOF does not by itself justify their ontological inclusion. DOF is introduced only when an observable phenomenon requires an irreducible relational structure beyond those already sufficient.

No observable phenomenon addressed in the present research requires a 3-DOF carrier. Therefore no 3-DOF or higher carrier is introduced at this stage.

This is not a claim that higher relational DOF do not exist. If future empirical observations require an additional irreducible relational degree of freedom, the carrier structure must be extended accordingly. Until such empirical necessity appears, introducing additional DOF would add structure without an identified physical relation and would violate Principle 1.4 and Principle 1.5.

Ontological Minimalism and Mathematical Transparency therefore retain S^1 and S^2 as the minimal relational carriers required by the presently expressed observable phenomena, without invoking metric geometry *a priori*. The possibility of additional carriers remains an open empirical question.

□

- 1-DOF: minimum irreducible relational structure required to encode a directional Change (kinematic)

- 2-DOF: minimum irreducible relational structure required to encode an isotropic omnidirectional Change (potential)
- 3-DOF+: not introduced unless empirical observations require an additional irreducible relational degree of freedom

WARNING: The Spatial Container Fallacy

S^1 and S^2 are not to be interpreted as physical geometries embedded in a spacetime container.

Do not visualise a sphere or a ring expanding through a 3D void.

- S^2 has no physical surface area, no volume, and does not "dilute" a flux across a spatial distance.
- S^1 and S^2 are algebraic state-space carriers encoding relational structures of the same conserved energy ledger.

Relational Carriers S^1 and S^2 = Logic of the Proportions

Spatial distance (r) and time (t) are emergent descriptive labels that observers attach to these underlying algebraic phase ratios. The phase projection dictates the space; the space does not dictate the phase.

Methodological constraints:	
$\downarrow (\text{Epistemic Hygiene} + \text{Relational Origin} + \text{Ontological Minimalism} + \text{Math Transparency}) \downarrow$	
Relational Closure + Causal Continuity + Isotropy Theorems	
$\downarrow (\text{apply to existing physics}) \downarrow$	
Standard Primitives:	$\underbrace{\text{background manifold} + \text{metric}}_{\text{spacetime structure (not derived)}} + \underbrace{\text{G, c, M, fields}}_{\text{energy dynamics (postulated)}}$
$\downarrow (\text{Ontological reduction by Relational Origin Principle} - \text{no background allowed}) \downarrow$	
One Primitive: $WILL \equiv SPACE-TIME-ENERGY \Rightarrow$ $SPACETIME \equiv ENERGY$	
$\downarrow (\text{by Relational Closure} + \text{Causal Continuity} + \text{Isotropy: derive relational energy carriers}) \downarrow$	
S^1 (1-DOF kinematic) + S^2 (2-DOF potential)	
$\downarrow (\text{by Duality of Relation Lemma}) \downarrow$	
Relational Conservation Theorem: $\underbrace{\text{Amplitude}^2}_{\text{External Transformation}} + \underbrace{\text{Phase}^2}_{\text{Internal Change}} = 1$	
$\underbrace{1/\beta_Y = 1 + z_\beta}_{\text{transverse Doppler shift}} \quad \underbrace{\beta^2 + \beta_Y^2 = 1}_{\text{Kinematic } S^1}$	Relational Projections $\underbrace{\kappa^2 + \kappa_X^2 = 1}_{\text{Potential } S^2} \quad \underbrace{1/\kappa_X = 1 + z_\kappa}_{\text{gravitational redshift}}$
$\downarrow (\text{by DOF-Indifference Lemma: equal quadratic weight to each independent DOF}) \downarrow$	
Closure Theorem: $\underbrace{1 \text{ Amplitude}^2}_{\text{on } S^2} = \underbrace{2 \text{ Amplitude}^2}_{\text{on } S^1} \Rightarrow$ $\kappa^2 = 2\beta^2$	
$\downarrow (\text{by Relational Conservation Theorem derive metric intervals as scaffolding}) \downarrow$	
Minkowski:	$\underbrace{\beta^2 + \beta_Y^2 = 1}_{\text{irreducible primitive}} + \underbrace{x, y, z, t \dots}_{\text{coordinate inflation}} = \underbrace{c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2}_{\text{mathematical scaffolding}}$
Schwarzschild:	$\underbrace{\kappa^2 + \kappa_X^2 = 1}_{\text{irreducible primitive}} + \underbrace{x, y, z, t \dots}_{\text{coordinate inflation}} = \underbrace{c^2 d\tau^2 = c^2 (1 - (R_s/r)) dt^2}_{\text{mathematical scaffolding}}$
$\downarrow (\text{by Relational Closure and Causal Continuity Theorems: any change must be balanced}) \downarrow$	
Energy Symmetry Law: $\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0$	
$\downarrow \Delta E_{A \rightarrow B} = E_0(\kappa_{X,B}/\beta_{Y,B} - \kappa_{X,A}/\beta_{Y,A}); \quad \Delta E_{B \rightarrow A} = E_0(\kappa_{X,A}/\beta_{Y,A} - \kappa_{X,B}/\beta_{Y,B}) \downarrow$	
Speed of light: $\beta \rightarrow 1 \rightarrow \beta_Y \rightarrow 0 \rightarrow E = E_0 \cdot \kappa_X / 0 \rightarrow \infty \rightarrow$ requires ∞ energy	
Event Horizon: $\kappa \rightarrow 1 \rightarrow \kappa_X \rightarrow 0 \rightarrow E = E_0 \cdot 0 / \beta_Y \rightarrow 0 \rightarrow$ requires 0 energy	
$\downarrow (\text{by Unified Relational Scaling Lemma}) \downarrow$	
Equivalence Principle $m_g \equiv m_i \equiv m = E_0/c^2$	
$\downarrow (\text{Lagrangian and Hamiltonian derived as collapsed single-point approximations}) \downarrow$	
$\underbrace{\Delta E_{A \rightarrow B} + \Delta E_{B \rightarrow A} = 0}_{\text{exact symmetry law}} \approx \underbrace{\frac{1}{2}(\kappa_A^2 - \kappa_B^2) + \frac{1}{2}(\beta_B^2 - \beta_A^2)}_{\text{first order approximation}} \approx \underbrace{T \pm V}_{\text{ontological collapse}}$	
$\downarrow (\text{apply RG to orbital mechanics}) \downarrow$	
Relational Orbital Mechanics R.O.M. : a closed algebraic system of equations	
$\downarrow (\text{by Factorization Theorem: every quantity } X \text{ splits into unitless core } \hat{X} \text{ and scale label } S_X) \downarrow$	
Unitless Core: $X = \underbrace{\hat{X}(\beta, e)}_{\text{unitless core}} \underbrace{S_X(R_s, c, G)}_{\text{units-scale}}$ G and M are unit-conversion factors.	
$\downarrow (\text{by Inverse Square, Closure and Causal Continuity Theorems}) \downarrow$	
No Singularities: $\beta_{\max}^2 = 1 \rightarrow \kappa_{\max}^2 = 2 \rightarrow r_{\min} = R_s / \kappa_{\max}^2 \rightarrow r_{\min} = R_s / 2$	
The point $r = 0$ is absent from the admissible relational range	
$\downarrow (\text{apply 2D to 3D translation interfaces to derive Relational Field Equation:}) \downarrow$	
$\kappa^2 = R_s / r = \rho_{\text{field}} / \rho_{\max}$ $\Rightarrow$ $\text{Spacetime Geometry}(R_s / r)$ $\equiv$ $\text{Energy Density}(\rho / \rho_{\max})$	
$\downarrow$ Theoretical Ouroboros $\downarrow$	
$SPACETIME \equiv ENERGY$ $\Leftrightarrow$ $SPACETIME GEOMETRY \equiv ENERGY DENSITY$	